Optimizing maintenance decisions in rails – a Markov Decision Process approach

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Abstract

Nowadays, railway transport is one of the most sustainable means of transportation, guaranteeing a quick movement of passengers and freights over short and medium distances with reduced usage of fossil fuels. The growing demand for this transport mode represents a major challenge to railway infrastructure managers in an attempt to guarantee cost-effective solutions without compromising the safety and reliability of railway infrastructures. The aim of the present dissertation is to provide an optimal decision map to support maintenance decisions in rail component. A Markov Decision Process (MDP) approach is followed to derive an optimal policy that minimizes the total costs over an infinite horizon depending on the different condition states of the rail. A practical example is explored with the estimation of the Markov Transition Matrices (MTMs) and the corresponding cost/reward vectors. The MTMs states are defined in terms of rail width, height, accumulated Million Gross Tons and damage occurrence. The optimal policy represents a condition-based maintenance plan with the aim of supporting railway infrastructure managers to take the best maintenance decision among a set of three possible actions depending on the state of the rail. The results obtained indicate that UIC60 rail profile requires that preventive maintenance actions are performed earlier than UIC54 rail profile.

Keywords: Railway maintenance, condition-based maintenance, optimizing maintenance, Markov Decision Process (MDP), wear, damage.

Resumo

Atualmente, o transporte ferroviário representa um dos mais sustentáveis meios de transporte, garantindo um transporte rápido de passageiros e mercadorias ao longo de pequenas e médias distâncias com uma utilização reduzida de combustíveis fósseis. Esta procura crescente representa um grande desafio para os gestores de infraestruturas ferroviárias numa tentativa de garantir soluções rentáveis do ponto de vista económico sem comprometer a segurança e a fiabilidade das infraestruturas ferroviárias. O objetivo desta dissertação é fornecer um mapa de decisão ótima para apoiar decisões de manutenção no carril. Um Processo de Decisão de Markov (MDP) é usado para calcular uma política ótima que minimize os custos totais ao longo de um horizonte infinito dependendo dos diferentes estados do carril. Um exemplo prático é explorado com a obtenção das Matrizes de Transição de Markov (MTMs) e os respetivos vetores de custo/remuneração. Os estados das MTMs são definidos em termos de largura de carril, altura, milhões de tonelagem bruta acumulada e ocorrência de dano. A política ótima representa um plano de manutenção com base na condição com o objetivo de ajudar os gestores de infraestruturas ferroviárias a tomar a melhor decisão de manutenção de entre um conjunto de três ações possíveis, dependendo do estado do carril. Os resultados obtidos indicam que o perfil UIC60 requer que as ações de manutenção preventiva sejam executadas mais cedo do que no perfil UIC54.

Palavras-chave: Manutenção ferroviária, manutenção com base na condição, otimização de manutenção, Processo de Decisão de Markov (MDP), desgaste, dano.

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Table of Contents

List of Figures

List of Tables

List of Acronyms

AIC	Akaike Information Criterion
AL	Alert limit
IAL	Immediate action limit
IL	Intervention limit
LM	Linear Model
MDP	Markov Decision Process
MGT	Million Gross Tons
MTM	Markov Transition Matrix
NDT	Non-destructive testing
RCF	Rolling contact fatigue
TQI	Track Quality Index

List of Symbols

Latin symbols	**Definition**
a	Action
C	Rail curvature
H	Rail height
I	Identity Matrix
MGT	Million Gross Tons
p_{cg}	MTM probabilities for the corrective grinding
p_{Damage}	Probability of transiting to a damaged state
p_{H54}	Probability of UIC54 rail profile decrease one interval in height from epoch *n* to epoch *n+1*
p_{H60}	Probability of UIC60 rail profile decrease one interval in height from epoch *n* to epoch *n+1*
p_{pg}	Probability of a one interval decrease in rail height (H) state as a result of a preventive grinding
p_{W54}	Probability of UIC54 rail profile decrease one interval in width from epoch *n* to epoch *n+1*
p_{W60}	Probability of UIC60 rail profile decrease one interval in width from epoch *n* to epoch *n+1*
P	Rail profile
P_1^{54}	MTM for the “Do Nothing” action ($a = 1$) for UIC54 rail profile
P_1^{60}	MTM for the “Do Nothing” action ($a = 1$) for UIC60 rail profile
P_2^{54}	MTM for the “Renewal” action ($a = 2$) for UIC54 rail profile
P_2^{60}	MTM for the “Renewal” action ($a = 2$) for UIC60 rail profile
P_3^{54}	MTM for the “Grinding” action ($a = 3$) for UIC54 rail profile
P_3^{60}	MTM for the “Grinding” action ($a = 3$) for UIC60 rail profile
P_{CG}	Sub-matrix of the corrective grinding for the “Grinding” action ($a = 3$)
P_{Damage}	Sub-matrix of the damage states for the “Do Nothing” action ($a = 1$)
P_{PG}	Sub-matrix of the preventive grinding for the “Grinding” action ($a = 3$)

P_{Wear}	Sub-matrix of the wear states for the “Do Nothing” action $(a = 1)$
$q^{1\ 54}$	Reward vector for the “Do Nothing” action $(a = 1)$ for UIC54 rail profile
$q^{1\ 60}$	Reward vector for the “Do Nothing” action $(a = 1)$ for UIC60 rail profile
$q^{2\ 54}$	Reward vector for the “Renewal” action $(a = 2)$ for UIC54 rail profile
$q^{2\ 60}$	Reward vector for the “Renewal” action $(a = 2)$ for UIC60 rail profile
$q^{3\ 54}$	Reward vector for the “Grinding” action $(a = 3)$ for UIC54 rail profile
$q^{3\ 60}$	Reward vector for the “Grinding” action $(a = 3)$ for UIC60 rail profile
RP	Rail relative position
W	Rail width

Greek symbols	**Definition**
γ	Discount factor
Δ	Variation
μ_{pg}	Average rail wear associated with a preventive grinding
μ	Mean value
σ	Standard deviation
$\hat{\mu}$	Estimated mean value
$\hat{\sigma}$	Estimated standard deviation

1. Introduction

This first chapter provides a historical background on railway industry, as well as the main objectives, the methodology used and the structure of this document.

1.1. Historical background

Railway industry played a key role in the transport sector in the 19th century during the First Industrial Revolution. The appearance of steam-powered engines on trains, more suitable for high speeds compared to other alternative means of transportation at the time, enabled quick transportation of passengers and freights [1]. The electrification of the railway network early in the 20th century represented a huge development in this sector enabling more powerful train engines, higher accelerations and longer life-spans compared to steam-powered engines, requiring less maintenance [2].

In the early days, mechanical engineers used do deal with vehicle dynamics, while railway infrastructure was typically assigned to civil engineers. The dynamic behaviour of rail vehicles along the track represented a major issue in the past. The strength of the materials and the adhesion between wheel and the rail was a challenge since dynamic loads applied were significantly high, leading to rail and wheelsets breakage and derailments. The guidance of the vehicles on track was also a problem which was solved with the adoption of the conical flanged wheels in the early 19th century.

The increasing demand for railway transportation requires accurate control of rail vehicle design and the vehicle dynamics on track is permanently tested using dynamic simulations. Railway track geometry and dimensioning as well as construction, inspection and maintenance procedures must comply with a set of technical standards. Nowadays, rail transport is characterized by the safety, comfort, low-cost usage and quick transportation of passengers, freights or other goods over short and medium distances. To fulfil these growing demands, a perfect interaction between rail, vehicle wheelsets and railway track must be ensured [1].

1.2. Objectives

With the increase in worldwide population, railway transport is becoming an even more relevant mean of transportation as an alternative to road and air vehicles. Climate changes and traffic demand are also increasing and it is very important to reduce the usage of fossil fuels and guarantee quick transportation of passengers and freights at the same time. To fulfil these growing needs it is important to improve reliability and reduce the life-cycle cost of railway infrastructures in terms of building and maintenance.

For railway infrastructure managers it is absolutely important to implement an adequate maintenance plan based on track geometry degradation predictive models in order to make the best maintenance decisions [3] to fulfil the needs of passengers and railway companies.

The main objective of the present dissertation is to create an optimal decision map in order to support maintenance decisions for the rail component in Portuguese railway lines using a condition-based maintenance policy which implies that maintenance actions are triggered by the actual condition of the rail. Within the scope of the present dissertation, the condition of the rail is defined in terms of rail width (W), height (H), accumulated Million Gross Tons (MGT) and damage occurrence and three possible actions can be performed after rail inspection: “Do Nothing”, “Renewal” and “Grinding”. Overall, the optimal decision map is obtained using a Markov Decision Process (MDP) approach in order to derive an optimal policy that minimizes the total costs over an infinite horizon.

1.3. Methodology

The methodology followed in the present dissertation took the following steps:

- Review theoretical background on Survival Analysis and MDP;
- Conduct data analysis from rail track inspections provided by the Portuguese railway infrastructure manager, especially information concerning rail wear;
- Conduct a statistical modelling approach to study the evolution of rail wear and assess the importance of the chosen rail profile wear variables in explaining the wear trajectories;
- Analyse rail wear and damage;
- Estimate the probability entries of Markov Transition Matrices (MTMs) and the corresponding reward vectors for different states and actions;
- Compute the optimal decision maintenance map for the rail component using the MDP approach.

1.4. Document structure

The present dissertation is structured in six chapters. In this first chapter, a historical background on railway industry, as well as the main objectives, the methodology used and the structure of this document are provided. Chapter 2 explores the main concepts behind a railway track, such as the interaction between vehicle and the track as well as the composition of a railway track, rail profile, track layout and track geometric parameters. Then, the technical aspects regarding the triad degradation, inspection and maintenance in the railway track are explored in chapter 3. In chapter 4, a theoretical background on Survival Analysis and MDP are provided as well as a literature review on the application of Markov models to support maintenance decisions in the railway industry. In chapter 5, a statistical model approach to study the evolution of rail wear is provided. In addition, a practical example based on data from the Portuguese railway lines is analysed with the estimation of the MTMs and the corresponding reward vectors based on rail degradation data with the aim of finding an optimal maintenance strategy for the Portuguese infrastructure manager, applying an MDP approach. Finally, in chapter 6, the main conclusions regarding the results obtained as well as the main assumptions and limitations of the model are mentioned. Some future paths for further research are also suggested.

2. Railway track specifications

This second chapter explores the main concepts behind a railway track, such as the interaction between vehicle and the track as well as the composition of a railway track. Rail profile, track layout and track geometric parameters are also addressed.

2.1. Vehicle-track interaction

As shown in Figure 2.1, a wheelset is composed of two conical wheels connected with a rigid axle. A wheel profile is composed of two surfaces: tread and flange. The tread rolls on the rail head during rail vehicle movement. The flange is responsible for the guidance of the vehicle along the track, counteracting the constant lateral movement of the wheelsets on the rail, preventing the occurrence of derailments. In a curve, rail vehicles are subjected to centrifugal forces. Normally, the rail is slightly inclined (1:20 or 1:40) to match the conical shape of the wheels.

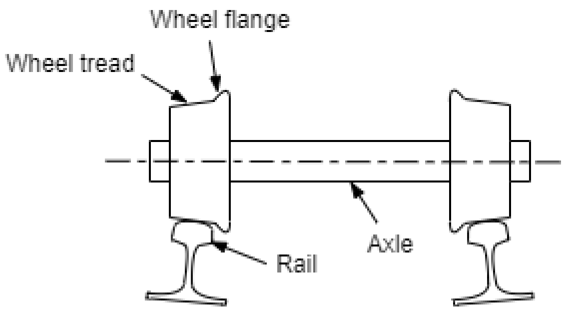


Figure 2.1 – Wheelset-rail interaction.

A railway track is responsible for supporting and guiding the vehicle along the track. The track is subjected to dynamic forces resultant from the contact between rails and wheelsets, which leads to the degradation of rail profiles and deviations in track geometric parameters and track layout, commonly known as track irregularities. The two wheels of a wheelset rotate at the same angular velocity. However, the lateral movement of the wheelsets allows different diameters to be in contact with the track when the rail vehicle is curving. The inner wheel is moving on the inner rail and the outer wheel is moving on the outer rail. In a curve, the outer wheel has to move a longer distance than the inner wheel which implies that the outer wheel has a higher linear velocity than the inner wheel (see Figure 2.2 (a)). On a straight track, the two wheels have the same rolling diameter and consequently the same linear velocity, unless some track irregularities or external forces resulting from higher speeds affect the natural movement of rail vehicle wheelsets (see Figure 2.2 (b)). This unstable running condition characterized by an oscillating motion around the track is called hunting [4]. Both sliding and rolling interactions occur in the contact zone.

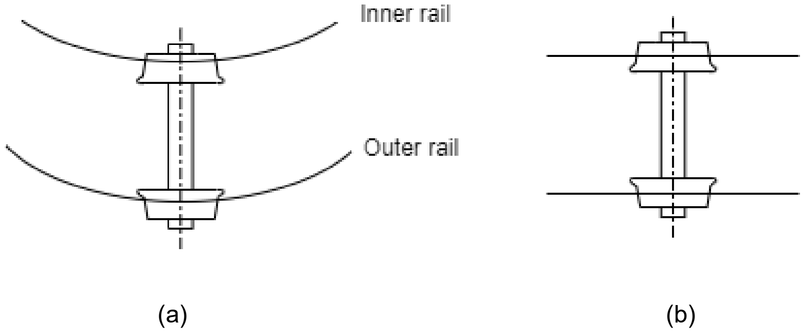


Figure 2.2 – Wheelsets position on track: (a) in a curve, (b) on a straight track.

2.2. Composition of a railway track

A railway track is composed of steel rails, sleepers, rail pads, fasteners, ballast, sub-ballast and subgrade [4] as shown in Figure 2.3. Rails are fastened to the sleepers and rail pads are placed between rails and sleepers. Rail pads stiffness plays a key role in vehicle dynamics since softer rail pads permit higher rail deflections and filter high-frequency vibrations transmitted to the sleepers. The sleepers are intended to transmit the loads from the rail to the ballast, which is tamped around the sleepers. The ballast is intended to support the track and is composed of coarse stones. The sub-ballast is composed of gravel and sand and is located between the ballast and the subgrade.

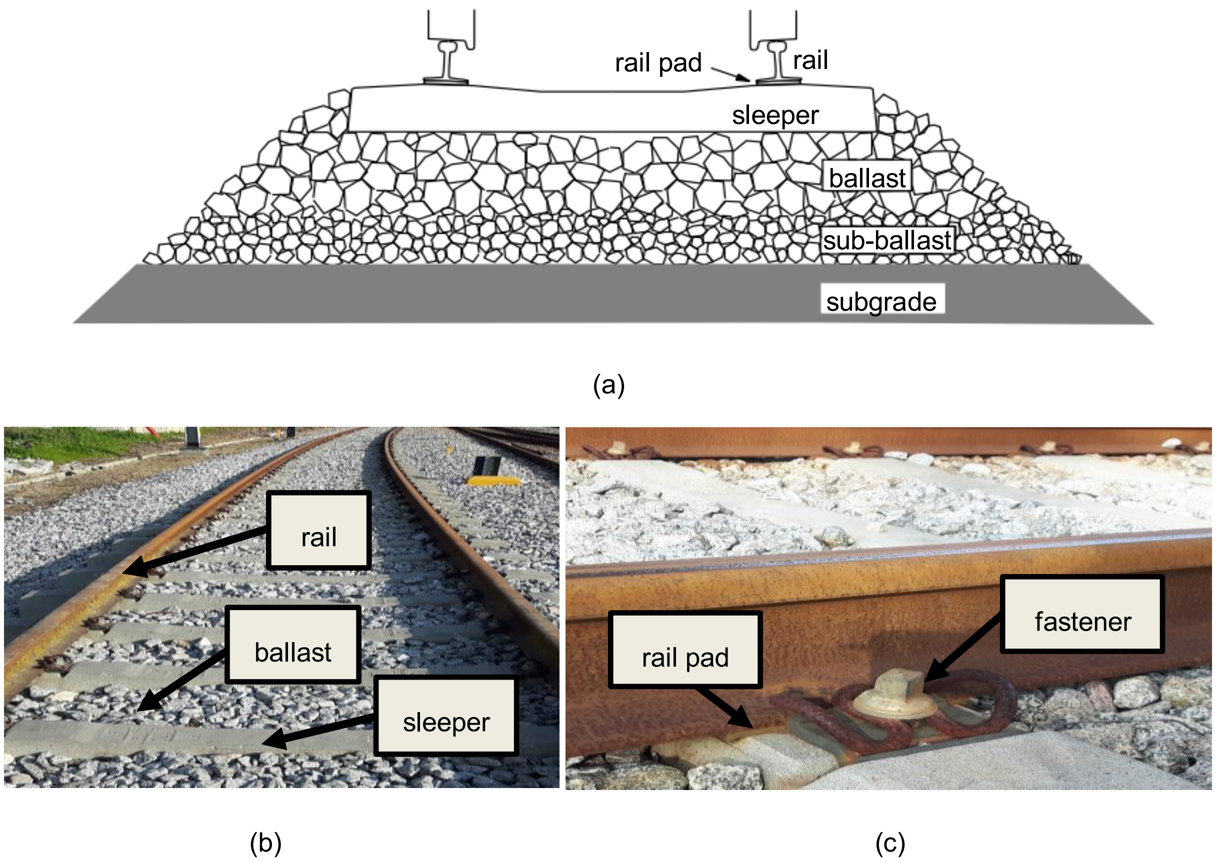


Figure 2.3 – (a) Schematic representation of the components of a railway track (adapted from [4]), (b) Railway track, (c) Railway track (detail).

2.3. Rail profile

As Figure 2.4 shows, rail profiles are composed of three parts: head, web and foot and are also characterised by several dimensions, being two of the most important:

- Height (H) – the linear distance between the two intersections of rail symmetry line with rail profile;

- Width (W) – the linear distance between the two intersections with rail head of a line located X mm below the top intersection of rail symmetry line with rail profile.

In the present dissertation, data from two rail profiles is used: UIC54 (54E1) and UIC60 (60E1). For the UIC54 rail profile, $X = 14.1$ mm, $W = 70$ mm and $H = 159$ mm. For the UIC60 rail profile, $X = 14.3$ mm, $W = 72$ mm and $H = 172$ mm. UIC54 and UIC60 rail profiles are manufactured and tested according to the European standard EN 13674-1 [5].

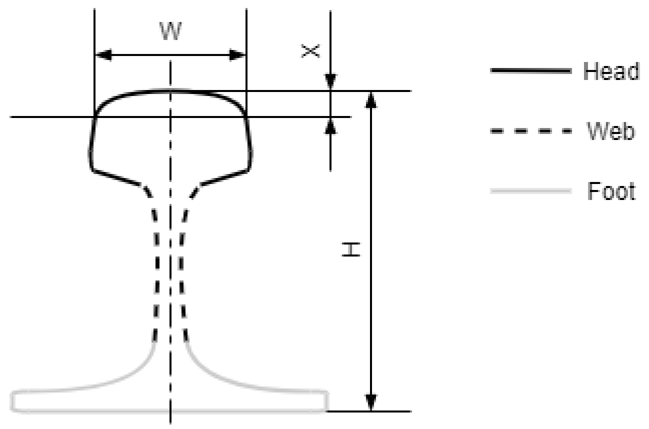


Figure 2.4 – Rail profile.

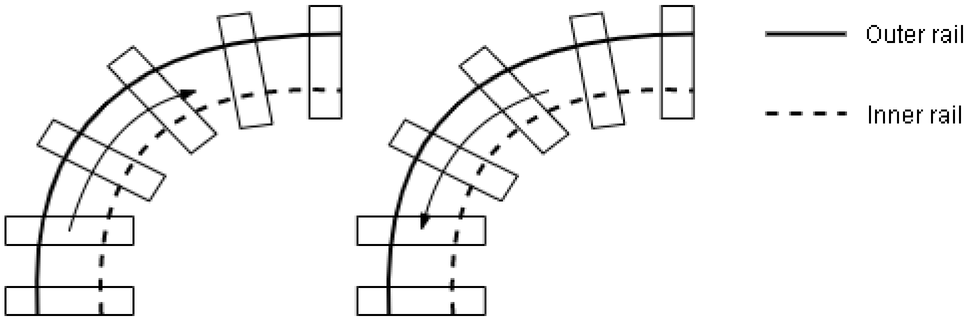
2.4. Track layout

The track layout is composed by straight tracks (which have an infinite radius (R)) and curves (which have a constant radius) being these two-track sections connected by transition curves (which have variable radius). An example of a track layout is represented in Figure 2.5. The curvature (C) is given as the inverse of the radius as represented in the following expression:

$$C = \frac{1}{R} \tag{2.1}$$

Figure 2.5 – Example of a track layout composed of straight tracks, curves and transition curves.

The curvature can either be positive or negative and the sign is related to the running direction and whether it is, respectively, a clockwise or counter-clockwise curve or transition curve as represented in Figure 2.6. On a straight track, the curvature value is zero as the radius is infinite.



(a) (b)

Figure 2.6 – (a) Positive curvature (clockwise curve or transition curve), (b) Negative curvature (counter-clockwise curve or transition curve).

Within the scope of the present dissertation, only the absolute value of rail curvature (C) is considered.

2.5. Track geometric parameters

The principal railway track geometric parameters are the gauge, cross level, twist, alignment and longitudinal level according to the European standard EN 13848-1 [6].

The gauge is the name given to the distance between the inner faces of the two rail heads, measured 14 mm below the running surface as shown in Figure 2.7. The European standard gauge is 1435 mm.

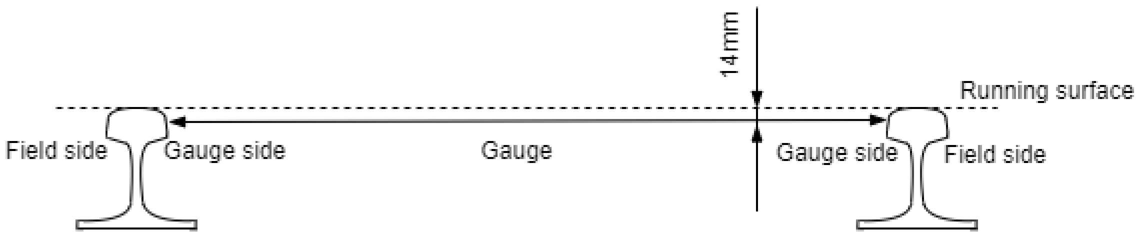


Figure 2.7 – Gauge.

In a curve or transition curve, the outer rail is usually higher than the inner rail to counteract the effect of centrifugal forces that tend to push the railway vehicle to the outer rail leading to asymmetric loadings. The cross level, also called cant or superelevation (see Figure 2.8), is the difference in height between the two rails. The cant angle, which is denoted by φ, is measured between the running surface and the horizontal plane.

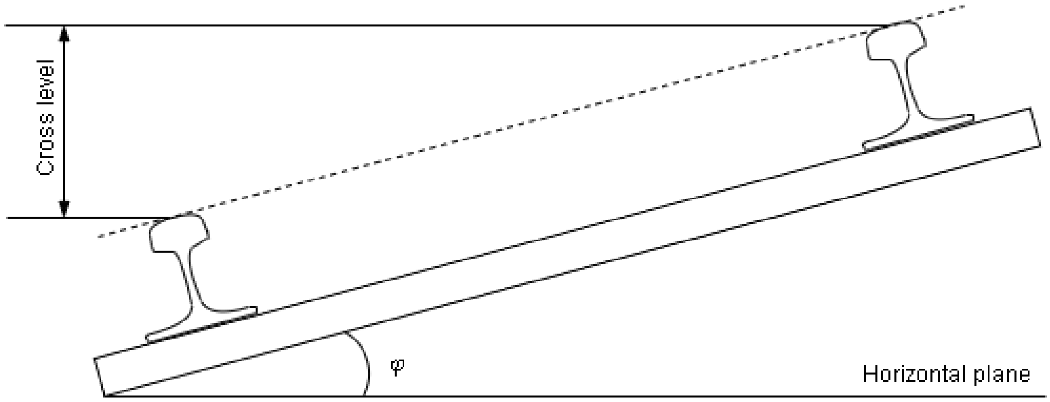


Figure 2.8 – Cross level.

The twist, usually expressed in mm/m, is the difference between two cross levels divided by their distance apart.

The alignment is the lateral deviation of the rail relative to the reference line and is denoted by AL_L and AL_R for the left and right rails, respectively, as shown in Figure 2.9.

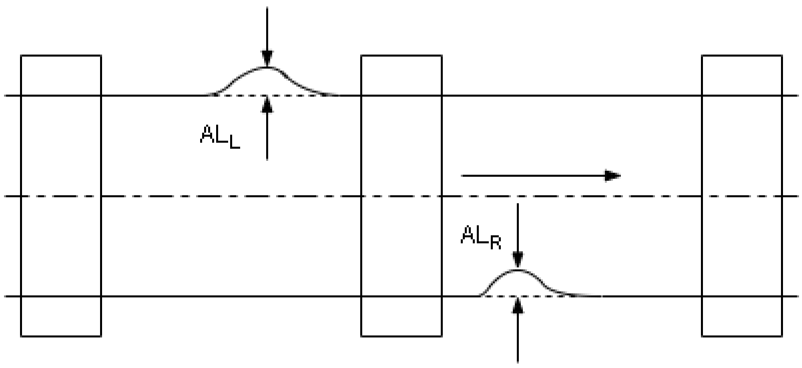


Figure 2.9 – Alignment.

The longitudinal level is the vertical deviation of the rail relative to the reference line and is denoted by LL_L and LL_R for the left and right rails, respectively, as shown in Figure 2.10.

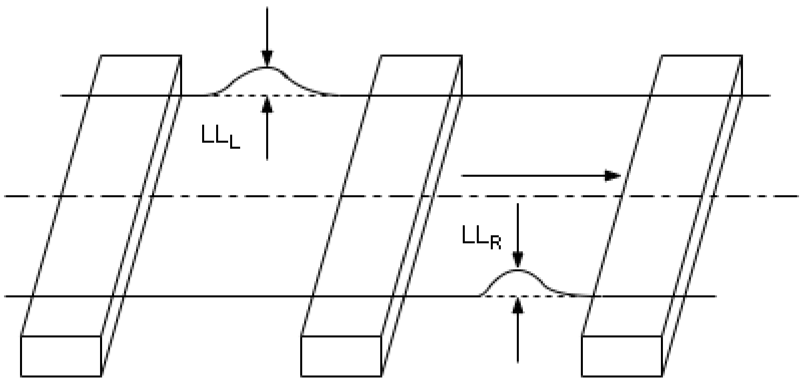


Figure 2.10 – Longitudinal level.

3. Degradation, inspection and maintenance in the railway track

In this chapter, the technical aspects regarding the triad degradation, inspection and maintenance in the railway track are explored. In section 3.1, the principal degradation mechanisms of the railway track are explained in terms of wear and damage. In section 3.2, the main procedures related to inspection activities in the railway track are presented. Finally, section 3.3 explores several maintenance actions.

3.1. Degradation

Rail profiles are continuously changing due to the loads and the high speeds that the rail is subjected during the passage of the vehicles, besides environmental conditions, such as the occurrence of precipitation or high temperature fluctuations (see Figure 3.1). When the rail is in service for a long time, such external factors become even more relevant to increase rail degradation process. High normal and lateral forces in the contact zone between wheelsets and rail tracks resultant from the traction and braking of rail vehicles may lead to yielding and fatigue of rail material. Consequently, vehicle dynamics are affected since track irregularities and worn rail profiles result in an increase in rail vehicle dynamic loads, vibrations and noise and may bring dangerous consequences such as derailments. High wear and damage propagation rates decrease rail life and consequently increase track maintenance costs as the rail has to be renewed more frequently [7]. Regular inspections are required to implement adequate maintenance activities in order to avoid damage propagation and reduce rail wear.

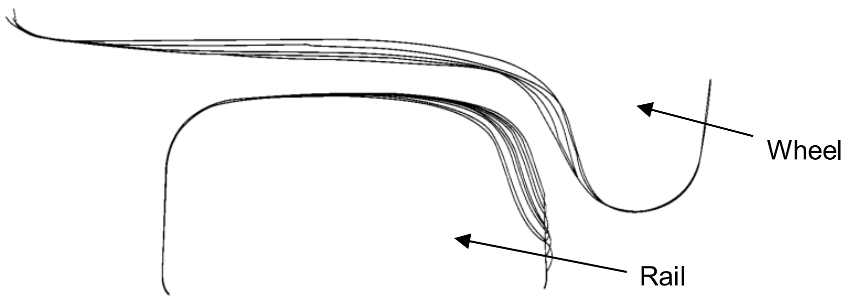
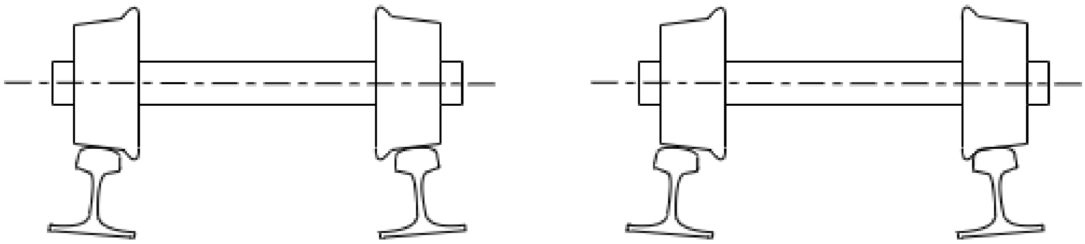


Figure 3.1 – Form of the change of wheel and rail from a Stockholm test case (adapted from [8]).

On a straight track, wheel treads are in contact with the top of the rail head and rolling interactions are more significant, typically producing lower wear rates than sliding interactions (see Figure 3.2 (a)). In a curve or transition curve, the wheel flange might be in contact with the gauge corner of the rail head and sliding interactions can become predominant (see Figure 3.2 (b)). Under these circumstances, the load is applied in a smaller area, which results in higher contact stresses, predominantly above the elastic limit, and consequent plastic deformation of the rail head leading to higher wear rates [4]. Normally, the outer rail is subjected to higher normal and tangential forces than the inner rail [8]. Higher values of track curvature would lead to higher wear rates of both inner and outer rails.



(a) (b)

Figure 3.2 – Contact between rail and wheelset: (a) on a straight track, (b) in a curve.

Besides wear, another important degradation/failure mechanism in rail profiles and the whole track is damage occurrence. As the demand for rail transportation is continuously increasing, issues regarding the contact between wheels and rails assume larger importance. Track irregularities increase the dynamic loads on the rail surface and rails are subjected to higher stresses which enhances the risk of crack initiation and propagation. Higher speeds and higher axle loads resultant from these increasing demands also promote the damage of the rails and of the whole track system, such as rail pads, fastenings, sleepers and ballast which can affect track stability. Wear rates have a strong influence on damage propagation since wear rates higher than crack propagation rates prevent cracks from propagating inside rail metal. On the other hand, high wear rates have a major impact on track degradation and are undesirable. Rail failures can have a considerable impact on the safety of passengers, resulting in vehicle delays and an increase in maintenance costs. Therefore, early detection of surface defects is important to prevent the occurrence of rail breaks [9]. According to EN 13231-5 [10], rolling contact fatigue (RCF), which results from the stresses' characteristics of the contact zone between wheels and rails, is the most common rail degradation/failure mechanism on European tracks and can appear in the form of several types of defects such as head check, belgrospi, squats, flaking and spalling (see Figure 3.3). However, other types of defects can occur such as flattened transverse profile, wheel burns, side cutting, lipping or corrugation.

(a) (b) (c)

Figure 3.3 – Types of defects resultant from RCF [10]: (a) Head check, (b) Squats, (c) Flaking.

3.2. Inspection

Due to railway track degradation, regular measurements of the track are required in order to detect either functional or safety failures. Track geometric parameters and rail profiles should be measured on a regular basis according to technical procedures defined in the European Standards. A track recording vehicle (see Figure 3.4) is equipped with two measuring systems: a track geometry measuring system and a rail profile measuring system.

Figure 3.4 – *EM-120* track recording vehicle.

A track geometry measuring system is a non-contact system intended to measure track layout parameters such as curvature and curve radius as well as track geometric parameters such as gauge, cross level, twist, alignment and longitudinal level (see Figure 3.5). It consists of:

- An inertial measurement unit (IMU) with accelerometers and gyroscopes;
- An optical gauge measuring system (OGMS);
- A navigation system with an integrated GPS receiver.

The IMU is an integrated sensor assembly consisting of 3-axis accelerometers and 3-axis gyroscopes. The spatial position of the measuring sensor is determined by double integration of acceleration measurements. The position accuracy of the IMU is improved by a navigation system with an integrated GPS receiver. OGMS is a laser measuring system intended to measure the displacement of each rail from a reference point, such as the centre of the IMU. The IMU and OGMS systems are mounted to the axles of the vehicle bogie.

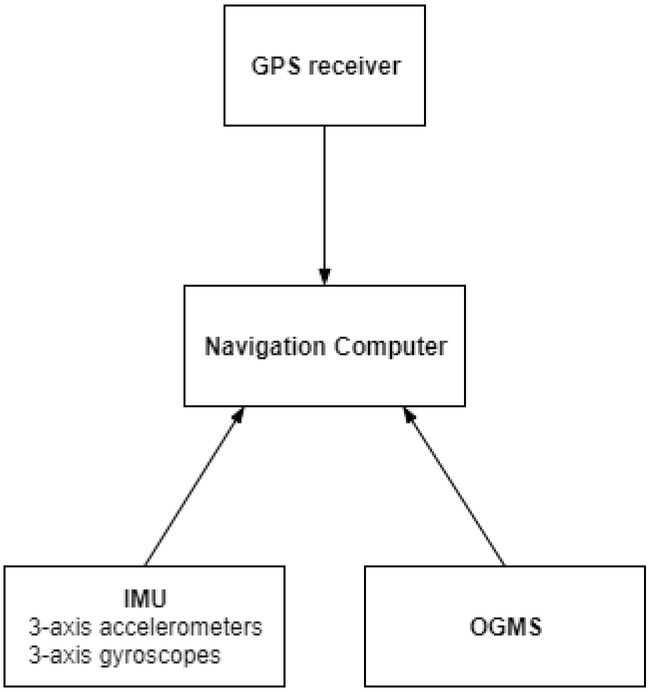


Figure 3.5 – Track geometry measuring system.

According to EN 13848-1 [6], track geometric parameters should be measured under loaded conditions of the track in order to simulate the dynamic behaviour of track vehicles. Therefore, the measuring sensors should be located as close as possible to one of the vehicle's loaded axles. When comparing different measurements of the same track geometric parameter performed on the same track section, several aspects related to the measuring conditions must be taken into account, such as the measuring speed, vehicle orientation or environmental conditions. EN 13848-5 [11] defines three different track geometric quality levels: alert limit (AL), intervention limit (IL) and immediate action limit (IAL). These limits are set for the different track geometric parameters depending on track speed ranges and wavelength measurement ranges. AL and IL reflect the most common practice adopted by railway infrastructure managers for a conscious maintenance strategy. IAL provides the highest acceptable value beyond which track quality and safety are seriously affected and immediate correction of track geometry is required. For more details on track geometry degradation and maintenance modelling, the reader is remitted to a recent literature review such as [12].

The rail profile measuring system consists of a laser measuring system which scans the rail at regular intervals defined by the operator. The cross section of the rail is hit by a light and the contours are recorded by a camera system. The original rail profile is automatically detected and a graphical display of the worn rail profile contour for left and right rails is provided as well as the rail head loss (in mm^2 and in percentage) in comparison with the original profile (see Figure 3.6). Using this procedure, the following dimensions are obtained:

- Lip – the amount of rail head extension beyond its normal cross section (in mm);
- Cant – the difference between the nominal and the measured inclination of the rail (in degrees);
- Height, Wear-Height – height and change in the rail height due to wear (in mm);
- Width, Wear-Width – width and change in the rail width due to wear (in mm).

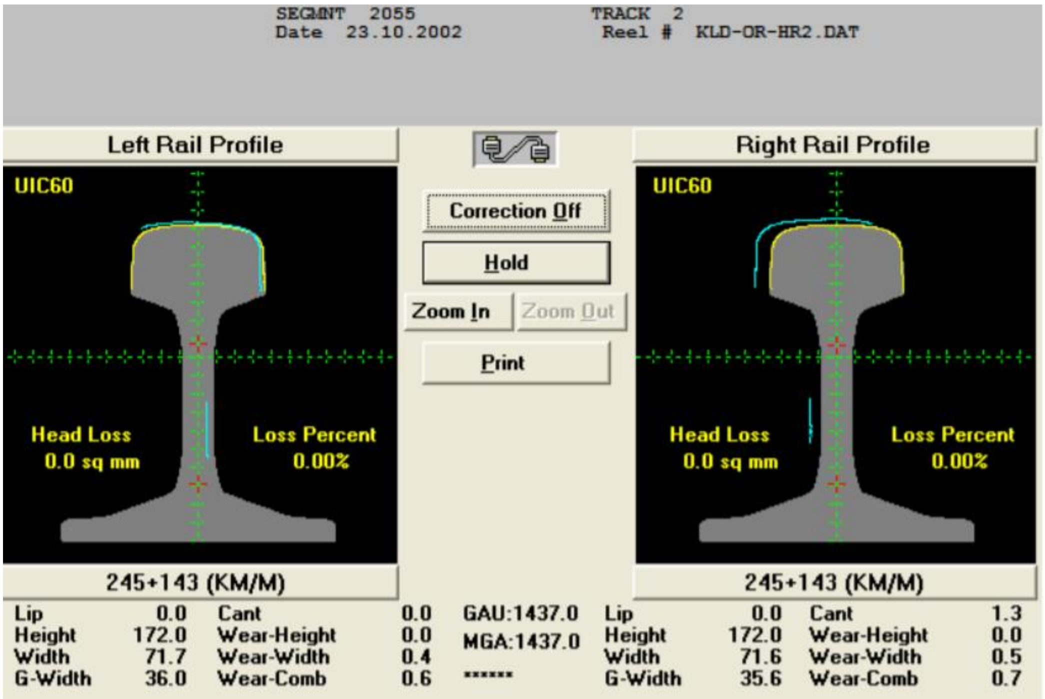


Figure 3.6 – Rail profile measurement display from Offboard software.

Rail profile wear must be within specified tolerance ranges defined in the Portuguese standard IT.VIA.021 [13]. Similar to the measurement of track geometric parameters, AL and IAL limits are defined depending on rail profile type and track speed ranges.

Non-destructive testing (NDT) is also a common practice within rail inspection procedures for the detection of either internal or external rail defects in order to ensure a safe railway operation and, thus, minimizing the occurrence of derailments. Visual testing and ultrasonic testing are the most common methods of defect detection in rails. Ultrasonic testing can be carried out either with hand equipment or using a special ultrasonic train.

The measured data related to the track geometric parameters, rail profile dimensions and damage occurrence must be analysed and assessed by railway infrastructure managers in order to carry out a set of adequate maintenance operations (see Figure 3.7).

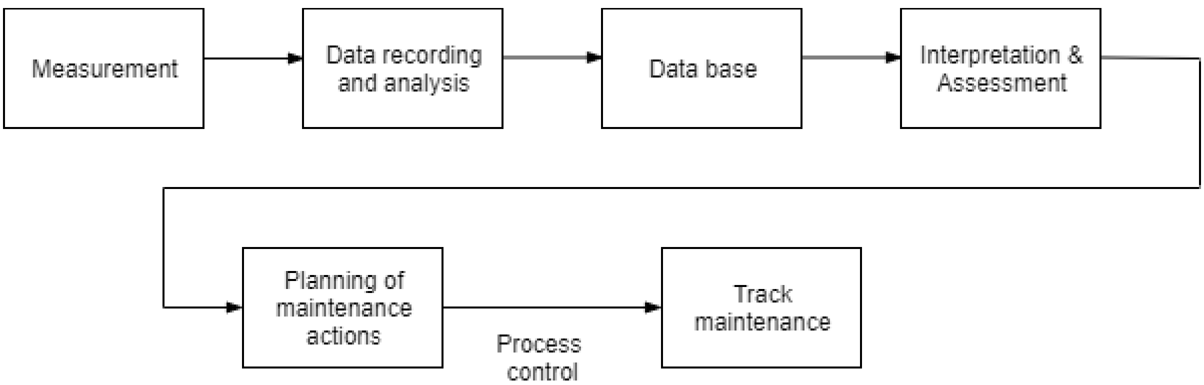


Figure 3.7 – Data analysis and subsequent maintenance actions.

3.3. Maintenance

Railway maintenance is a set of operations carried out by railway infrastructure managers in order to keep railway infrastructures and equipment in a good, reliable and safe operational condition according to several quality and safety standards at minimum cost. It involves correcting, improving or replacing the affected components, always with the vision of maximum durability, efficiency and cost-effectiveness in mind. Railway maintenance can be distinguished in two main types such as:

- Preventive – planned maintenance actions performed time-to-time and on a regular basis to detect possible component failures;
- Corrective – unplanned maintenance actions performed in order to repair the component to the required operational condition after damage or failure occurrence.

Another maintenance strategy can be adopted such as predictive maintenance. This maintenance type is intended to detect changes in the physical condition of the components to carry out the appropriate maintenance actions and can be based on condition monitoring (condition-based) or statistical-data for predicting failure or damage occurrence (statistical-based).

Railway components degrade with usage and age and in order to increase their reliability and anticipate expectable failures or damage occurrence, a good maintenance strategy is required to be performed on a regular basis. The costs inherent to adequate and planned maintenance scheduling are much less significant than the additional costs associated with the complete failure of railway components and the required corrective maintenance activities [14]. Track maintenance involves several activities such as rail reprofiling, tamping or track renewal and must comply with a set of technical standards.

Rail reprofiling (grinding)

Rail reprofiling consists in removing surface defects such as corrugations or track irregularities resultant from rail vehicle operation or small manufacturing defects and is useful to maintain wheel-rail contact conditions at an acceptable level. These defects are removed through the rotational movement of the grinding stones and by continuous monitoring and adjustment of stone inclination, pressure and grinding speed as shown in Figure 3.8. Thus, any specified target profiles can be achieved within the required tolerance ranges.

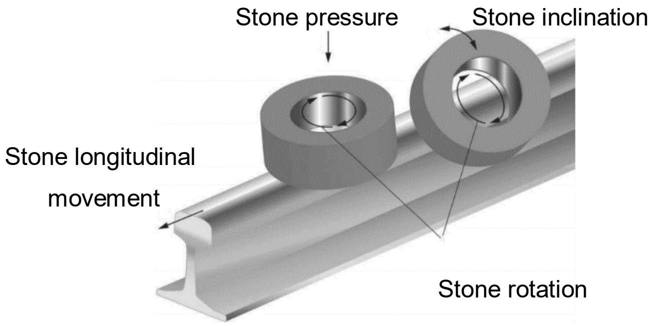


Figure 3.8 – Grinding reprofiling (adapted from [10]).

Several aspects need to be considered when reprofiling, such as the amount of metal removal necessary to eliminate or reduce the defect, the desired shape of the rail profile after reprofiling (target profile) and the evenness of the longitudinal profile [15]. Rail reprofiling strategy can be divided into three different types:

- Initial – small defects resultant from the rail manufacturing or installation are removed and the rail head profile is optimized;
- Preventive – regular reprofiling in which small defects resultant from the rail vehicle operation on the railway track are removed in order to maintain wheel-rail contact in a good condition;
- Corrective – removal of severe defects such as corrugations or other track irregularities caused by high speeds or axle loads inherent to an intensive rail vehicle operation on track.

The target profile must contain the area where wheel-rail contact occurs, which usually ranges between the -70º tangent at the gauge side and the 5º tangent at the field side as shown in Figure 3.9.

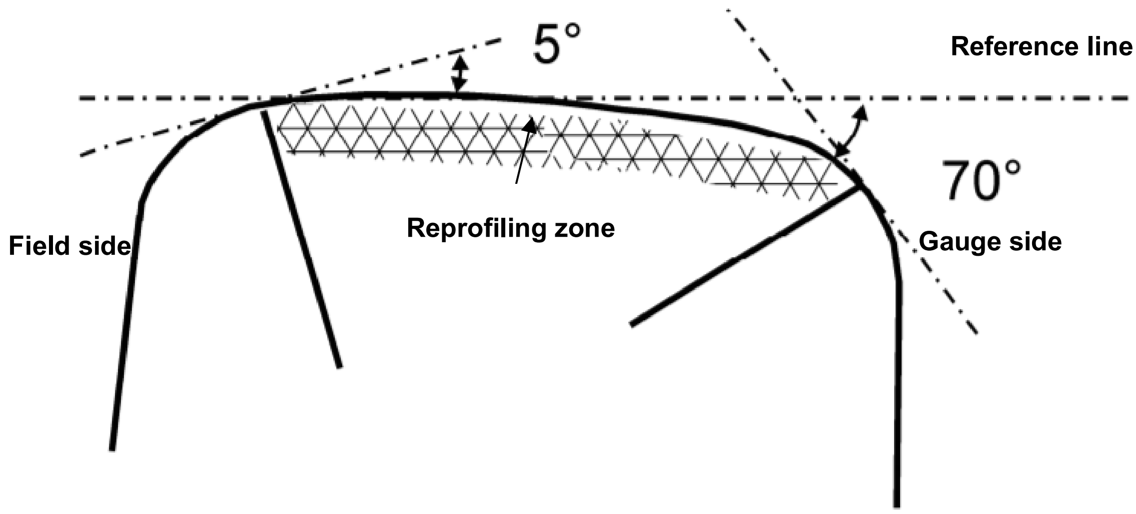


Figure 3.9 – Rail reprofiling zone (adapted from [16]).

Regarding rail reprofiling, some tolerance requirements must be taken into account by railway infrastructure managers and whether tight or wide product tolerances are more suitable for railway operational condition. Tighter tolerances result in higher durability of the reprofiled rail, although higher production costs are associated. Wider tolerances lead to lower production costs and thus higher productivity, despite the fact that more regular maintenance activities are required. These tolerances represent the range of deviations of the measured reprofiled rail from the target profile and can either be positive or negative. To carry out these measurements, the reprofiled rail must be aligned with the target profile. According to EN 13231-3 [16], three different tolerance classes are specified for rail reprofiling. In the example shown in Figure 3.10, the measured reprofiled rail is below the target profile, which represents a negative deviation.

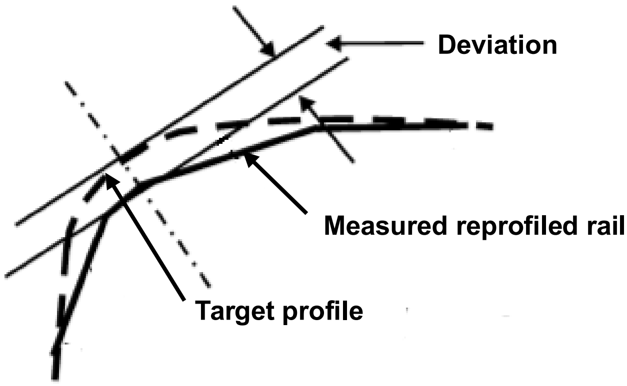


Figure 3.10 – Negative deviation of the measured reprofiled rail from the target profile specified by railway infrastructure managers (adapted from [16]).

Tamping

Tamping is a maintenance procedure for railway track intended to improve the geometric quality and track stability. As mentioned previously, these irregularities increase rail vehicle dynamic loads on the rail surface and also play an important role in rail profile degradation. The tamping machine is positioned over the sleepers and lifts the track up to the target vertical or horizontal position [17]. Two hydraulic non-synchronous tamping tines working at constant pressure are used to squeeze the ballast under the sleepers. The rotating movement of an eccentric shaft creates a linear vibration in the squeeze direction and facilitates the penetration of the tamping tines on the ballast, supporting the ballast consolidation and increasing the railway track stability (see Figure 3.11).

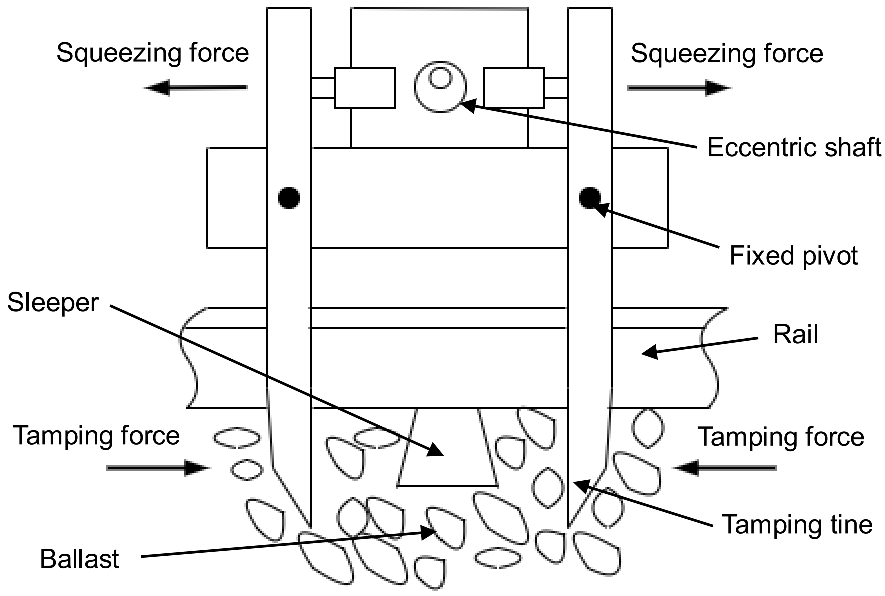


Figure 3.11 – Schematic representation of a tamping machine.

The working on track ballast, namely track dynamic stability, ballast compaction and cleaning, is done in compliance with EN 13231-1 [18]. For instance, the ideal squeeze time ranges between 1 and 1.2 s. The maximum track lift, working speed and working depth of tamping tools are dependent on the track construction and must be set by the entity in charge of maintenance. Ballast thickness, inclination and arrangement must be within tolerance ranges also specified in this technical standard.

Track renewal

A track renewal should occur when rail dimensions or track geometric parameters are out of the standardized tolerances. Several aspects are also taken into account such as the quality of the sleepers, fasteners, rail pads as well as the actual and future associated maintenance costs. In the case of partial track renewal, some additional technical aspects related to the uniformity of the track must be considered. Track sections ranging between 5 and 10 km must be renewed at a time to guarantee a homogeneous track quality and a perfect interaction between track structural components [2].

4. Survival Analysis and Markov Decision Process

In this fourth chapter, the main theoretical concepts behind this dissertation are explored. In section 4.1, a theoretical background on Survival Analysis is provided, since this type of approach will be further used for estimating transition probabilities in the MDP. In section 4.2, the main principles related to MDP are presented, including a theoretical background on Markov chains and MDP, since MDP approaches provide an ideal framework for condition-based maintenance and deal with cases of prediction of future states and optimization of maintenance policies. Finally, a literature review on the application of Markov models on maintenance decisions in the railway industry is also provided in section 4.2.

4.1. Survival Analysis

A Survival Analysis is a collection of methods for analysing data where the outcome variable is the time until an event of interest occurs, normally a component failure, being that time called "survival time" [19].

Reliability is the probability that a component can perform its service functions longer than a specific period of time (called "survival time") under service conditions. Denoting the survival time of a component by T and the specific time value for that variable by t, the reliability function $R(t)$ represents the probability that the variable T exceeds any given value of time *t* as written in the equation below:

$$R(t) = \mathrm{P}(T > t) \tag{4.1}$$

A reliability function is a decreasing curve that ranges from $t = 0$, where $\mathrm{R}(0) = 1$ to $t = \infty$, where $R(\infty) \approx 0$ as shown in Figure 4.1.

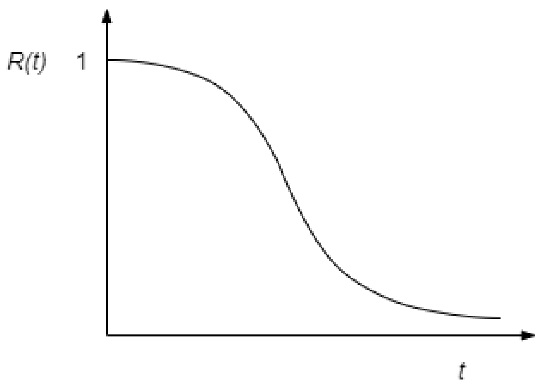


Figure 4.1 – Theoretical reliability function.

$F(t)$ represents the probability of failure of a component until or at the instant of time *t* and is given by:

$$F(t) = 1 - R(t) \tag{4.2}$$

The time to failure distribution $f(t)$ represents the probability of failure of the component at the instant of time *t* and is obtained by derivation of expression (4.2).

$$f(t) = F'(t) = -R'(t) \tag{4.3}$$

The time to failure distribution $f(t)$ can be modelled as a Weibull distribution, where β is the shape parameter and η is the scale parameter of the distribution:

$$f(t) = \frac{\beta}{\eta}(\frac{t}{\eta})^{\beta-1}e^{-(\frac{t}{\eta})^{\eta}} \tag{4.4}$$

The probability that the component survives until t_f $R(t_f)$ is given by:

$$R(t_f) = \int_{t_f}^{+\infty} f(t)dt \tag{4.5}$$

The hazard rate $h(t)$ represents the instantaneous potential per unit of time that the failure of the component occurs at instant time *t,* given that no failure occurred until time *t:*

$$h(t) = \frac{f(t)}{R(t)} = -\frac{R'(t)}{R(t)} \tag{4.6}$$

The cumulative hazard rate $H(t)$ is obtained by integrating $h(t)$ over time:

$$H(t) = \int_0^t h(t)dt = -\ln(R(t)) \tag{4.7}$$

4.2. Markov Decision Process

Markov Decision Process is a model for sequential decision making, involving decision epochs, states, actions, rewards and transition probabilities. Its aim is to make the best decision (action) in each state in order to generate the best reward and that same decision will also determine the next state through a transition probability function [20].

4.2.1. Markov chains

The Markov chain model has been created by Andrey Markov (1856-1922), a Russian mathematician and professor at St. Petersburg University in Russia. It consists of a random (or stochastic) sequence of states equally spaced in points of time called epochs (n) (see Figure 4.2). The time between each epoch is called period or step. At each epoch n, a random vector of states X_n represents the probability of being in each one of the states (s) with $s \in \{1,2,3,\dots,N\}$. The sum of the entries of the vector X_n is 1 since the chain is in any of the states at each epoch [21].

$$X_n = [P(s = 1)\ \ P(s = 2)\ \ P(s = 3) \dots P(s = N)] \tag{4.8}$$

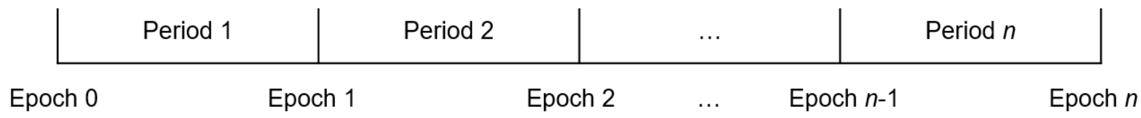


Figure 4.2 – Chronological sequence of periods and epochs.

The transition probabilities for a Markov chain represent a one-step or one-period transition probability and are organized in a $N \times N$ square matrix where N represents the number of possible states. This square matrix is called Markov Transition Matrix (MTM). An MTM is composed of a set of conditional probabilities that state the probability p_{ij} that the chain is in state *j* at epoch $n + 1$, given that it is in state *i* at epoch n. It can be written by:

$$p_{ij} = P(X_{n+1} = j | X_n = i) \tag{4.9}$$

Each row of an MTM represents the present state at epoch n and each column represents the next state at epoch $n + 1$. An example of an MTM for a chain with N states is represented below. The sum of all probabilities p_{ij} of each row is equal to 1 since that if the chain is in state *i* at epoch *n*, it is certain that the chain is in any of the possible states at epoch $n + 1$. Each MTM has non-negative entries and no entries greater than 1 [21].

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1N} \\ p_{21} & p_{22} & \cdots & p_{2\mathrm{N}} \\ \vdots & \vdots & \ddots & \vdots \\ p_{N1} & p_{N2} & \dots & p_{NN} \end{bmatrix} \tag{4.10}$$

Let the chain be described by a vector of states X_n at epoch n. Then, using the MTM, denoted by P, the vector of states X_{n+1} is obtained for epoch $n + 1$ using the following expression:

$$X_{n+1} = X_n \cdot P \tag{4.11}$$

Given a chain described by an initial vector of states X_0 and an MTM denoted by P, the vector of states X_n after n epochs is obtained applying the following expression:

$$X_n = X_0 \cdot P^n \tag{4.12}$$

The transition probabilities and the respective Markov chain can be represented graphically and an example of a three-state Markov chain with $s \in \{1,2,3\}$ is shown in Figure 4.3.

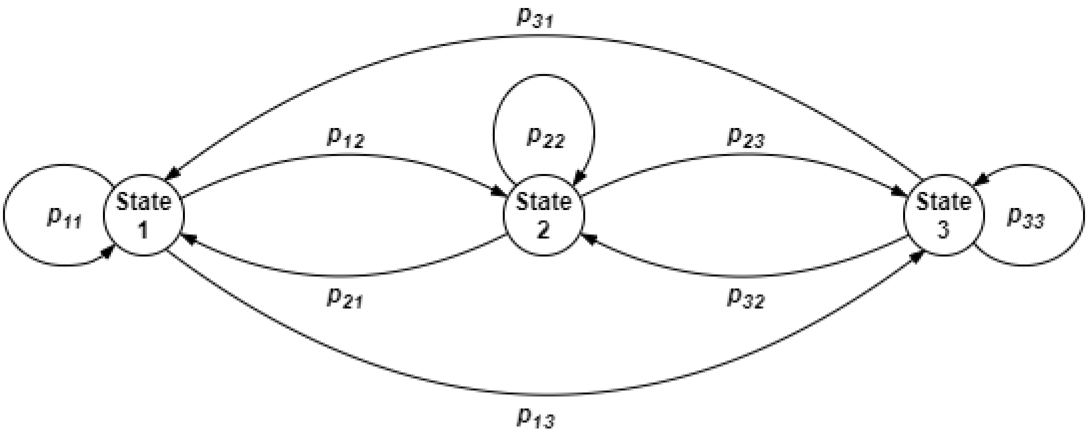


Figure 4.3 – Graphical representation of a three-state Markov chain and transition probabilities.

4.2.2. Decision process

As previously mentioned in this chapter, an MDP is a sequential decision-making process. It involves taking the best decision or action from a finite set of actions $a \in \{1,2, \dots A\}$, for each state the chain is with $s \in \{1,2,3, \dots, N\}$. The main objective is to create a set of decisions for each state, called policy, in order to maximize the sum of all rewards [20].

The MTM for each one of those actions can be denoted by P^k and is composed of a set of conditional probabilities that state the probability p_{ij}^k that the chain is in state j at epoch $n+1$, given that it is in state i at epoch n and action k is taken. It can be written by:

$$p_{ij}^k = P(X_{n+1} = j | X_n = i \cap a_i = k) \tag{4.13}$$

The reward vector q_i is a vector whose entries represent the immediate rewards earned at the end of each epoch by visiting state i and taking a specific action. In the majority of the cases, immediate rewards are assumed to be stationary over time. This means they do not depend on the epoch they are earned, but only on the state the chain is and the action taken [21].

$$q_i = [q_1 \;\; q_2 \;\; q_3 \dots q_N]^T \tag{4.14}$$

The vector of expected rewards R earned after n steps is given by:

$$R = P^n \cdot q \tag{4.15}$$

Considering a finite horizon of T epochs, the sum of expected rewards earned from epoch 0 until epoch T is given by vector $v(0)$. Vector $v(T)$ represents the ultimate rewards earned if the process ends in final state i at epoch T, commonly known as salvage values.

$$v(0) = P^0 \cdot q + P^1 \cdot q + \dots + P^{T-1} \cdot q + P^T \cdot v(T) \tag{4.16}$$

In Sheskin [21], a recursive backward solution called value iteration is used to obtain an optimal policy which maximizes the total rewards $v(n)$ earned from epoch n until the end of the planning horizon T given that the chain is in state i at epoch n. The discount factor $\gamma \in [0,1]$, which is considered in value iteration, is a scale factor that represents the difference in importance between the present and the future rewards received.

$$v_i(n) = \max_k \left[q_i^k + \gamma \sum_{j=1}^{N} p_{ij}^k v_j(n+1) \right] \tag{4.17}$$

for $n = 0,1, \dots, T-1$ and $i = 1,2, \dots, N$

To start the iterative process, a vector of salvage values $v_j(T)$, received at the end of epoch T must be specified. This vector represents the rewards earned in every state j the chain can be at the end of epoch T.

The optimal policy obtained is given by a decision vector $d(n)$ that defines the best action to take at each epoch n given that the chain is in state i at epoch n.

$$d(n) = [d_1(n) \;\; d_2(n) \dots d_N(n)]^T \tag{4.18}$$

For an infinite horizon, decision vector d obtained is stationary over time which implies that it will always specify the same action depending on the state the chain is, regardless of the epoch. The solution can be obtained using four different computational methods: value iteration, exhaustive enumeration, policy iteration and linear programming.

Although value iteration requires fewer arithmetic operations than the alternative computational methods, the main limitation regarding this method for an infinite horizon is that stopping criteria may never be satisfied. Exhaustive enumeration is feasible only for small state spaces since it involves listing every possible policy and determining which one maximizes the total rewards for every possible state. Policy iteration consists of choosing an initial random policy and iterating it successively until it converges to the optimal policy. The method used in the present dissertation is linear programming since MDP problems can be solved by using computer software toolboxes, containing linear programming functions such as *mdp_LP* function available in a MATLAB toolbox [22].

Markov models are widely used by engineers and managers in several research fields such as meteorology, gambling, chemistry, marketing, economics, healthcare, communication networks and inspection/maintenance activities. Although MDP approaches can provide optimal policies for several practical applications, the main limitations regarding this method are that some unrealistic assumptions have to be made. For the particular case of maintenance planning, the measurement errors inherent to inspection activities represent a source of uncertainty when evaluating the real state of the structures. However, there are some researchers using alternative methods that account for the presence of measurement uncertainty in inspection activities. This was achieved by Madanat et al. [23] and Madanat [24] using the Latent MDP in their research works, which is an extension of the traditional MDP.

4.2.3. Markov models applied in railway maintenance

The increasing demand for railway transportation, mostly over the last decade, has been a challenge for railway companies in an attempt to guarantee reliable and cost-effective maintenance strategies for railway infrastructures and rolling stock [25]. To fulfil these needs, some researchers have been evaluating and developing several maintenance strategies using Markov models based on degradation data of wheel and the railway track.

Regarding metropolitan train wheels, Jiang et al. [25] presented a semi-MDP approach considering wear in terms of flange diameter and thickness based on periodic condition monitoring and determined an optimal reprofiling policy that minimizes the maintenance cost per unit of time expected in the long run. Mingcheng et al. [26] applied the same approach to high-speed train wheels, adding damage data to the model. Braga and Andrade [27] developed an MDP approach to model wear and damage occurrence on railway wheelsets, providing an optimal decision map that, depending on the wheelset diameter, mileage since last turning (or renewal) and damage occurrence, supports the decision-maker to make the best choice among a set of three possible actions: “Do Nothing”, “Renewal” and “Turning”.

Relatively to railway infrastructure, Prescott and Andrews [28] described a Markov model to study the influence of different track maintenance strategies in the change of track section condition with time for British rail network rated between 80 and 110 miles/hour. The track section condition is based on the standard deviation of longitudinal level and is divided into four classifications: “good condition”, “maintenance requested”, “speed restriction required” and “line closure required”. Bearing in mind the actual and the previous condition of the track section inspected at regular intervals of time so as the number of tamping actions since renewal, the Markov chain to predict track degradation is divided into 80 states. By varying model parameters such as inspection interval, renewal period and mean time to perform routine maintenance, some results related to the probability that the track section is in each of the four conditions throughout its lifetime are obtained. This model can be used to evaluate the risk of potential derailments based on the actual condition of the track.

Brkic and Adamovich [29] suggested a model to evaluate the reliability and safety of railway system by analysing operational data from “Serbian Railways” subsystems such as station relay devices, power supply equipment, station insulated sections or station signals. Regarding this research study, a Markov model with four different states (“the failureless state”, “the state with partial failures”, “the completely failed state” and “the blocked state”) is presented. By varying some parameters such as failure rate and repair rate, system states can be predicted and railway operation and maintenance is optimized.

Zakeri and Shahriari [30] developed a probabilistic model to predict rail condition using data from rail wear in curved tracks with 250 meters radius of Lorestan railway. Rail condition index (γ) is based on the relationship between average lateral wear amount and maximum allowable lateral wear according to general Islamic Republic Railway of Iran standards and three Markov states are defined (“good condition”, “average condition” and “poor condition”) depending on the rail condition index. This model is useful to predict rail condition at any time and decide whether a renewal action should be taken. It must occur when “poor condition” is the state with the highest probability.

Bai et al. [31] predicted the evolution of track irregularities in Chinese railways using a Markov chain that describes the evolution of the track quality. The states are discretized in four levels of the TQI, which is widely used to describe track geometric parameters. This Markov model is based on periodic measurements of track irregularities using track inspection vehicles.

An optimal maintenance decision plan for Iranian railways was obtained by Shafahi and Hakhamaneneshi [32] considering a planning horizon of 10 years. A Markov chain is used to predict track deterioration and the ability of the track to perform its function, which is described by the TQI. The TQI varies from 0 to 100 and is divided into five states on the Markov chain ("failed", "medium", "good", "very good" and "excellent"). Tracks are divided into six classes according to traffic density (light and heavy) and topography condition (plain, hilly and mountainous areas). An MDP approach is used to obtain an optimal maintenance plan containing three possible actions ("Routine Maintenance", "Improvement" and "Reconstruction") for each class of the track, which improves the accuracy of this model.

Sharma et al. [33] provided an MDP optimal maintenance policy based on data collected from a Class I railroad in North America considering three possible actions ("Major Maintenance", "Minor Maintenance" and "No Maintenance"). A Markov chain divided into five states is used to model wear evolution based on the TQI and the occurrence of geo-defects. A lower value of the TQI implies higher probabilities of geo-defect occurrence. The average costs associated with the maintenance of 1 mile of track for a period of 10 years are compared considering the optimal policy and the existing policy and the savings are estimated for three different inspection intervals.

Table 4.1 summarizes these research works concerning maintenance strategies in the railway industry using Markov approaches.

Table 4.1 – Previous research works concerning maintenance strategies in the railway industry using Markov approaches.

Reference	Variables considered	Scope of the research
Jiang et al. [25]	Wheel flange diameter and thickness	Semi-MDP approach to optimize reprofiling policy in metropolitan train wheels
Mingcheng et al. [26]	Wheel flange diameter, thickness and external shocks	Semi-MDP approach to optimize reprofiling policy in high-speed train wheels
Braga and Andrade [27]	Wheel diameter, mileage since last turning and damage occurrence	MDP approach to optimize maintenance decisions in railway wheelsets
Prescott and Andrews [28]	Standard deviation of track longitudinal level	Markov model to study the influence of inspection interval, renewal period and mean time to perform routine maintenance in track section condition
Brkic and Adamovich [29]	Railway subsystems such as station relay devices or station signals	Markov model to predict the evolution of railway subsystems in terms of failure rate and repair rate

Zakeri and Shahriari [30]	Rail lateral wear	Markov model to study rail degradation in a curved track of 250 m and decide whether a renewal action is needed
Bai et al. [31]	Track Quality Index (TQI)	Markov model to predict the evolution of track irregularities in different track sections
Shafahi and Hakhamaneneshi [32]	Track Quality Index (TQI)	MDP approach to optimize maintenance decisions in railways for different track conditions considering a planning horizon of 10 years
Sharma et al. [33]	Track Quality Index (TQI) and geo-defects occurrence	MDP approach to optimize maintenance decisions in a railway track. The average costs associated with the maintenance of 1 mile of track are also compared considering the optimal policy and the existing policy for a period of 10 years

Overall, the previous studies presented for optimizing maintenance actions tend to rely on knowledge-based inspections or seem to be short on quantitative wear variables for characterizing the wear level of the rails. None of the previous studies has considered wear in terms of rail profile height and width dimensions, neither included the accumulated Million Gross Tons (MGT) of traffic that have passed over the rail as a variable in Markov chain states. Moreover, the "Grinding" action is not considered in any of the previous studies regarding optimal maintenance policies for railways using an MDP. In fact, rail grinding can be either used for removing damaged material layers or extending the life-cycle of the rail components [34], and should, therefore, be taken into account in the present dissertation. Cuervo et al. [35] concluded that there is a good correlation between the number of grinding operations and the types of defects or rail profile degradation mechanisms that can appear in rails.

In the next chapter, a practical example of railway maintenance is explored for Portuguese railway lines, using an MDP approach to model wear evolution, in which the state space is defined in terms of rail height, width, accumulated MGT and damage occurrence. The contribution of the present dissertation is to extend the previous models, which are very often subjective in nature and rely on knowledge-based inspection, towards more objective MDP approach, with quantified geometric indicators defining the condition and state of the rail component.

5. Application

In this fifth chapter, the main steps to estimate the probabilities of the Markov Transition Matrices (MTMs) are provided, as well as an MDP application for this practical case. The problem is divided into three possible actions that can be performed after rail inspection: “Do Nothing”, “Renewal” and “Grinding”. This implies that three MTMs and three reward vectors should be defined, in order to generate an optimal maintenance strategy for Portuguese railway lines. Since two rail profiles (UIC54 and UIC60) are considered in this practical case, two optimal maintenance strategies will be obtained. In section 5.1, an exploratory statistical model approach to study the evolution of rail wear is presented for both change in the rail height due to wear (ΔH) and change in the rail width due to wear (ΔW), as a function of several explanatory variables. In section 5.2, the state space is defined for the two UIC54 and UIC60 rail profiles. In sections 5.3, 5.4 and 5.5, MTMs are obtained for “Do Nothing”, “Renewal” and “Grinding” actions, respectively. In section 5.6, three reward vectors are obtained for each of the two UIC54 and UIC60 rail profiles. In section 5.7 two optimal policy maps are provided.

5.1. Statistical modelling of the evolution of rail wear

In the first stage, a statistical exploratory analysis to assess the data provided by the Portuguese railway infrastructure company is performed. This data was captured in 2007, 2008 and 2009 inspections that were made in “Linha de Cintura”, in order to measure the height (H) and width (W) values of the rail profiles in 8389 track kilometric points. With the measured rail height (H) and width (W) values, the change in the rail height due to wear (ΔH) and the change in the rail width due to wear (ΔW) values are obtained. Information about rail profile (UIC54 or UIC60), rail relative position (RP) (upper (U), lower (L) or zero-cross level (Z)) and rail curvature (C) for each of the 8389 track kilometric points is also provided.

The second stage consists of data analysis from “Linha de Cintura” track information in order to determine the year of renewal of each of the 8389 track kilometric points. Then, the data mentioned previously is matched with the rail lifetime of each specific track kilometric point at the time of inspection. An annual average traffic value of 8 MGT was assumed for “Linha de Cintura”.

The data analysed was obtained from inspections that occurred in 2007, 2008 and 2009 in five different months: March 2007, March 2008, December 2008, March 2009 and October 2009. The assumption made is that, for instance, rail height (H) and width (W) values measured in a track kilometric point renewed in 1980, in an inspection occurred in March 2007 correspond to a rail lifetime of 27.25 years (218 MGT) as well as the data obtained for a rail renewed in 1970 and inspected in October 2009 correspond to a rail service life of 39.8 years (318.4 MGT). Since information about rail month of renewal is not provided, it is assumed that it occurs at the beginning of the respective year.

The variables mentioned previously are summarized in Table 5.1.

Table 5.1 – Descriptive statistics of the main variables in rail wear (mean, minimum and maximum values for 8389 kilometric points).

Variables	Description	Type	Mean	Min	Max
ΔH	Change in the rail height due to wear [mm]	Continuous	2.765324	0	11.800000
ΔW	Change in the rail width due to wear [mm]	Continuous	0.183943	0	7.900000
MGT	Million Gross Tons [MGT]	Continuous	182.612000	42.000000	326.400000
H	Rail height [mm]	Continuous	162.667300	148.900000	172.000000
W	Rail width [mm]	Continuous	70.804970	62.100000	72.000000
C	Rail curvature [m^{-1}]	Continuous	0.001267	0	0.002870
RP	Rail relative position (3 types: upper, lower or zero-cross level)	Nominal	-	-	-
P	Rail profile (2 types: UIC54, UIC60)	Nominal	-	-	-

- **Change in the rail height due to wear (ΔH)**

The values of change in the rail height due to wear (ΔH) as a function of the accumulated MGT and the correspondent polynomial regression are represented in Figure 5.1.

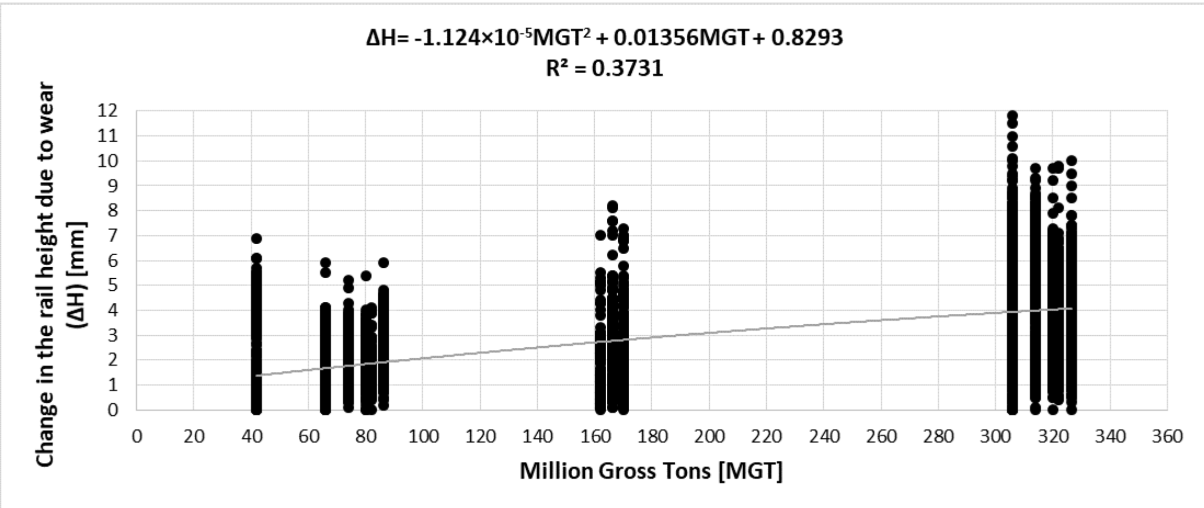


Figure 5.1 – Change in the rail height due to wear (ΔH).

There is a lot of variability in the values of change in rail height due to wear (ΔH) around the polynomial regression, which is not explained by the accumulated MGT. This variability can be explained by several variables such as rail profile (P), rail relative position (RP) and rail curvature (C) using a Linear Model (LM), treating them as fixed effects. The results of the analysis of the change in rail height due to wear (ΔH) are presented in Table 5.2. Five models (M0-M4) were tested for the dependent variable ΔH being the model M0 the simplest one considering only the fixed effect MGT and containing an intercept, a linear and a quadratic parameter. In models M1, M2 and M3, the fixed effects of rail curvature (C), rail relative position (RP) and rail profile (P) are added successively and in Model M4 an interaction term between rail curvature and rail relative position ($C \times RP$) is added to Model M3. The Akaike Information Criterion (AIC) value is used to compare the different model specifications.

Table 5.2 – Estimates for the parameters of models M0-M4 for the dependent variable "Change in the rail height due to wear (ΔH)".

Model label	Parameter	M0 - ΔH	M1 - ΔH	M2 - ΔH	M3 - ΔH	M4 - ΔH
Fixed effects						
1	β_0 (a)	0.8293 (0.07766)	0.8171 (0.07859)	0.6442 (0.0846)	0.2807 (0.1518)	0.6355 (0.1542)
MGT	β_{MGT} (a)	0.01356 (0.001262)	0.01337 (0.001274)	0.01357 (0.001217)	0.01595 (0.00147)	0.01615 (0.00146)
MGT^2	β_{MGT^2} (a)	-1.124×10^{-5} (3.271×10^{-6})	-1.075×10^{-5} (3.307×10^{-6})	-1.187×10^{-5} (3.161×10^{-6})	-1.562×10^{-5} (3.417×10^{-6})	-1.606×10^{-5} (3.393×10^{-6})
C	β_C (a)	-	17.1 (16.80) (b)	-104.3 (23.15)	-115.8 (23.48)	-345.5 (31.42)
RP	β_U	-	-	0	0	0
	β_L (a)	-	-	0.9465 (0.0344)	0.9484 (0.03439)	0.1628 (0.07974)
	β_Z (a)	-	-	0.03132 (0.05584) (b)	0.02544 (0.05585) (b)	-0.3497 (0.06527)
P	β_{54}	-	-	-	0	0
	β_{60} (a)	-	-	-	0.2496 (0.08654)	0.2598 (0.08594)
$C \times RP$	$\beta_{C\times U}$	-	-	-	-	0
	$\beta_{C\times L}$ (a)	-	-	-	-	483.8 (44.38)
	$\beta_{C\times Z}$	-	-	-	-	0
Scale						
σ		1.452	1.452	1.386	1.385	1.376
AIC						
		30069.64	30070.61	29292.37	29286.05	29169.89
Number of parameters						
		4	5	7	8	9

(a) Standard errors (b) Non-significant coefficient (*p*-value higher than 0.05)

Model M4 is the one which best explains the variability in the values of change in rail height due to wear (ΔH) around the second-order polynomial regression and is mathematically defined as:

$$\Delta H = \beta_0 + \beta_{MGT} \cdot MGT + \beta_{MGT^2} \cdot MGT^2 + \beta_C \cdot C + \beta_{RP} \cdot RP + \beta_P \cdot P + \beta_{C\times RP} \cdot (C \times RP) \quad (5.1)$$

- **Change in the rail width due to wear ($\varDelta W$)**

The values of change in the rail width due to wear ($\varDelta W$) as a function of the accumulated MGT and the correspondent polynomial regression are represented in Figure 5.2.

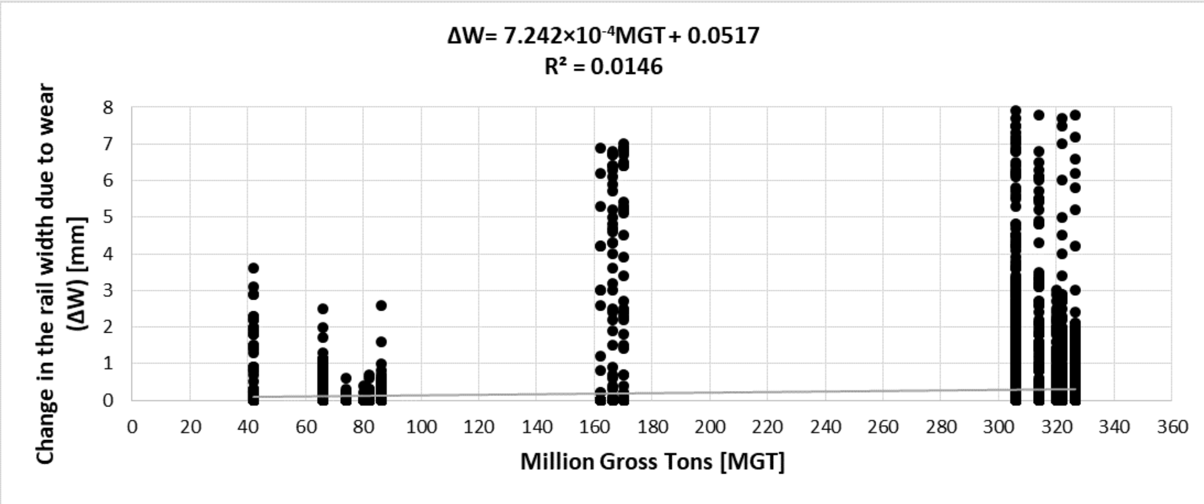


Figure 5.2 – Change in the rail width due to wear ($\varDelta W$).

There is also a lot of variability in the values of change in rail width due to wear ($\varDelta W$) around the polynomial regression, which is not explained by the accumulated MGT. This variability can be explained by the same variables presented for the previous case, excluding rail profile (see Table 5.3).

Table 5.3 – Estimates for the parameters of models M0-M3 for the dependent variable "Change in the rail width due to wear ($\varDelta W$)".

Model label	Parameter	M0 - $\varDelta W$	M1 - $\varDelta W$	M2 - $\varDelta W$	M3 - $\varDelta W$
Fixed effects					
1	$\boldsymbol{\beta_0}$ (a)	0.0517 (0.01424)	0.05198 (0.01811)	0.2146 (0.02819)	0.2626 (0.03336)
$\boldsymbol{MGT}$	$\boldsymbol{\beta_{MGT}}$ (a)	7.242×10^{-4} (6.502×10^{-4})	7.24×10^{-4} (6.515×10^{-5})	6.714×10^{-4} (6.679×10^{-5})	6.707×10^{-4} (6.677×10^{-5})
$\boldsymbol{C}$	$\boldsymbol{\beta_C}$ (a)	-	-0.2032 (8.249) (b)	-25.31 (11.8)	-54.56 (16.05)
$\boldsymbol{RP}$	$\boldsymbol{\beta_U}$	-	-	0	0
	$\boldsymbol{\beta_L}$ (a)	-	-	-0.2269 (0.01771)	-0.3272 (0.04134)
	$\boldsymbol{\beta_Z}$ (a)	-	-	-0.1815 (0.02871)	-0.2293 (0.03378)
$\boldsymbol{C \times RP}$	$\boldsymbol{\beta_{C\times U}}$	-	-	-	0
	$\boldsymbol{\beta_{C\times L}}$ (a)	-	-	-	61.81 (23.01)
	$\boldsymbol{\beta_{C\times Z}}$	-	-	-	0
Scale					
$\boldsymbol{\sigma}$		0.7207	0.7208	0.7136	0.7134
AIC					
		18316.53	18318.53	18152.95	18147.73
Number of parameters					
		3	4	6	7

(a) Standard errors (b) Non-significant coefficient (p-value higher than 0.05)

Model M3 is the one which best explains the variability in the values of change in rail width due to wear (ΔW) around the linear polynomial regression and is mathematically defined as:

$$\Delta W = \beta_0 + \beta_{MGT} \cdot MGT + \beta_C \cdot C + \beta_{RP} \cdot RP + \beta_{C \times RP} \cdot (C \times RP) \quad (5.2)$$

The fixed effect associated with rail profile (P) was excluded from the statistical modelling of the change in the rail width due to wear (ΔW), mainly because the obtained estimates for the models containing rail profile (P) as a fixed effect did not provide satisfactory results, as ΔW seemed to decrease with the increase of accumulated MGT, which is not reasonable.

These two studies represent a statistical approach to analyse the evolutions of the changes due to wear in the rail height (ΔH) and in the rail width (ΔW), as a function of several explanatory variables. Bearing in mind that several more variables not considered in this analysis can explain the variability of ΔH and ΔW around the polynomial regression (e.g. track topography, range of speeds or braking and traction zones), some limitations are obviously associated to this statistical modelling approach.

Regarding the principal scope of the present dissertation, which is optimizing maintenance decisions in rails, only the evolution of the change in ΔH and ΔW as a function of the accumulated MGT will be considered in the estimation of the MTM probabilities for the two rail profiles (P), without taking into account the other explanatory variables used for these two studies such as rail relative position (RP) or rail curvature (C). This is mainly due to the need to make MDP formulation simpler. Nevertheless, it is left for further research to investigate how to integrate all these relevant dependencies in a next MDP approach.

5.2. Definition of MDP state space

The MDP state space is defined as a combination of four variables: rail width (W), height (H), accumulated MGT and damage occurrence. Rail width (W) and height (H) variables are grouped in 1 mm intervals, assuming a maximum and minimum value. The minimum values considered for rail profile width (W) and height (H) variables for UIC54 and UIC60 rail profiles are defined according to the AL wear values for track speeds less or equal than 80 km/h specified in IT.VIA.021 [13]. The maximum values are defined according to the initial width (W) and height (H) dimensions for UIC54 and UIC60 rail profiles. The states in which width (W) and/or height (H) reach their minimum interval are called scrap states. Thus, the rail is considered to be in a scrap state when the minimum interval in width (W), in height (H) or in both width (W) and height (H) states is reached. Moreover, the damage variable only assumes two nominal values: “with damage” or “without damage”; whereas the accumulated MGT variable is discretized in steps of 8 MGT, from 0 MGT to 352 MGT.

The summary of the MDP state space for UIC54 and UIC60 rail profiles is presented in Table 5.4. However, the detailed definition of MDP state space for UIC54 and UIC60 rail profiles is extremely important for a better understanding of this problem (see Appendices A1 and A2, respectively).

Table 5.4 – Summary of the MDP state space for UIC54 and UIC60 rail profiles.

		UIC54 rail profile			UIC60 rail profile		
Variable	**Precision/ step**	**Minimum**	**Maximum**	**Number of states**	**Minimum**	**Maximum**	**Number of states**
Width (W)	1 mm	57 mm	70 mm	13	56 mm	72 mm	16
Height (H)	1 mm	145 mm	159 mm	14	156 mm	172 mm	16
Accumulated MGT	8 MGT	0 MGT	352 MGT	45	0 MGT	352 MGT	45
Damage	-	-	-	182	-	-	256
Total number of states	-	8 372			11 776		

5.3. MTM for the "Do Nothing" action ($a = 1$)

The "Do Nothing" action ($a = 1$) consists in assuming that the rail is in an acceptable condition and able to continue in service, so letting it degrade without any maintenance or renewal action performed.

For the "Do Nothing" action ($a = 1$), the problem is divided into two separate main analysis: wear and damage analysis and each one of them is fundamental to estimate the MTMs probabilities. In subsection 5.3.1, the probabilities obtained in the wear analysis depend on the profile analysed (UIC54 or UIC60), which means that they assume different values in UIC54 and UIC60 rail profiles MTMs. In the subsection 5.3.2, the probabilities obtained in the damage analysis are independent of the profile, which means that they assume the same values for the two UIC54 and UIC60 rail profiles MTMs for the "Do Nothing" action.

5.3.1. Wear analysis

The wear analysis is based on the statistical modelling conducted in section 5.1. The values of change in the rail width due to wear (ΔW) and change in the rail height due to wear (ΔH) are plotted as a function of the accumulated MGT. These two graphical representations are made for the two UIC54 and UIC60 rail profiles. The respective polynomial regression functions, which are imposed to intersect the origin, are shown for each graphical representation. The values of the change in the rail width due to wear (ΔW) and change in the rail height due to wear (ΔH) as a function of the accumulated MGT for UIC54 rail profile are represented, respectively, in Figures 5.3 and 5.4. The values of the change in the rail width due to wear (ΔW) and change in the rail height due to wear (ΔH) as a function of the accumulated MGT for UIC60 rail profile are represented, respectively, in Figures 5.5 and 5.6.

The probabilities that, at any given state, rail profiles width (W) or height (H) states decrease one interval are derived by assuming only neighbouring state transitions are possible. For instance, a UIC54 rail profile can only decrease 1 mm in width (W) every 8 MGT with a certain transition probability p_{W54} or remain with the same width (W) with probability $1 - p_{W54}$. Then, the expected width wear in each next 8 MGT can be expressed through the Markovian concept as the sum of the multiplication of the possible wear variations and their corresponding transition probabilities. For example, since the considered possible wear variations are no wear variation (0 mm) or 1 mm of wear variation, the expected wear variation for a UIC54 rail profile after 8 MGT in its width is computed as $p_{W54} \cdot (1) + (1 - p_{W54}) \cdot (0)$. By making this expected value equal to the mean wear after 8 MGT estimated through the values given by the regression obtained in Figure 5.3 ($\Delta W(MGT + 8) - \Delta W(MGT)$), the expression (5.3) is obtained. The same is applicable for UIC54 rail profile height (H) and UIC60 rail profile width (W) and height (H). As mentioned in section 5.2, the accumulated MGT variable is discretized in steps of 8 MGT, from 0 MGT to 352 MGT. The probabilities that rail width (W) or height (H) states of UIC54 and UIC60 rail profiles decrease one interval from epoch *n* to epoch *n+1* (the step between epochs is 8 MGT) depend only on the rail accumulated MGT at epoch *n* and are independent of the width (W) or height (H) values of the rail at epoch *n*.

- **UIC54 rail profile**

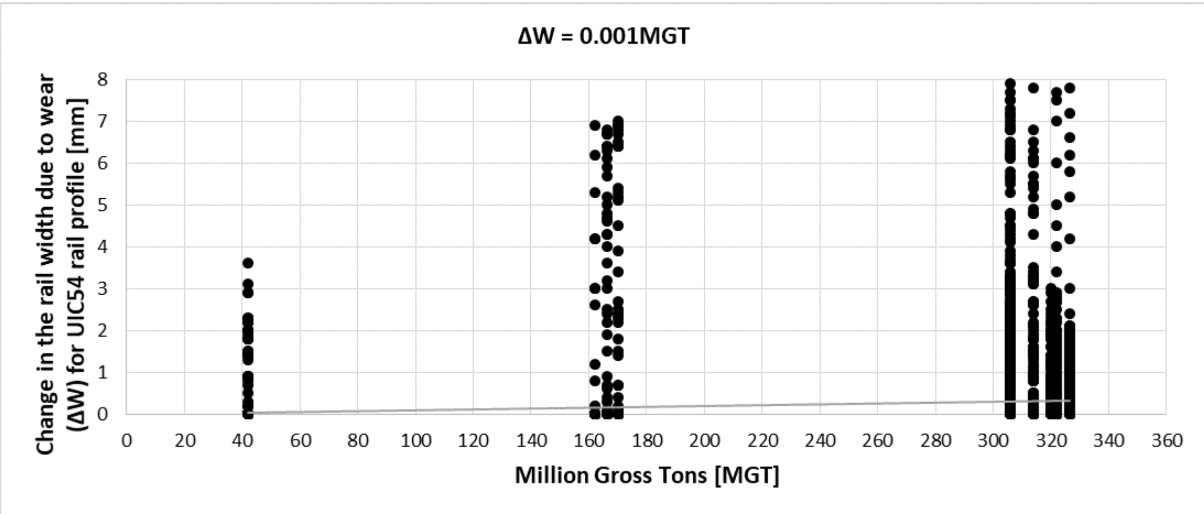


Figure 5.3 – Change in the rail width due to wear (ΔW) for UIC54 rail profile.

$$p_{W54}(MGT) = \frac{\Delta W(MGT + 8) - \Delta W(MGT)}{1} = 0.008, MGT = 0, 8, 16, 24, \dots, 344 \quad (5.3)$$

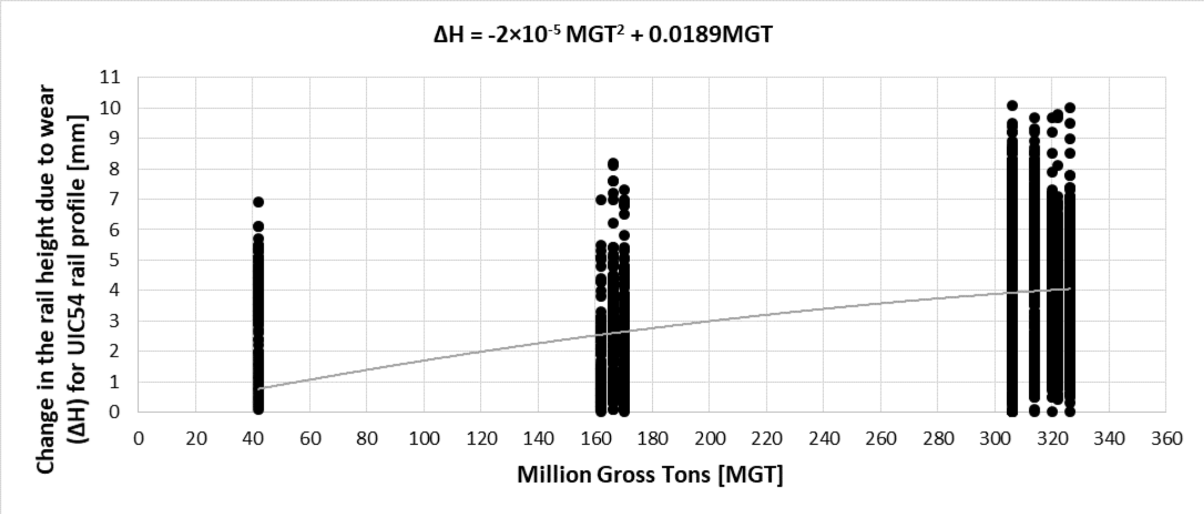


Figure 5.4 – Change in the rail height due to wear (ΔH) for UIC54 rail profile.

$$p_{H54}(MGT) = \frac{\Delta H(MGT + 8) - \Delta H(MGT)}{1}, MGT = 0, 8, 16, 24, \dots, 344 \quad (5.4)$$

- **UIC60 rail profile**

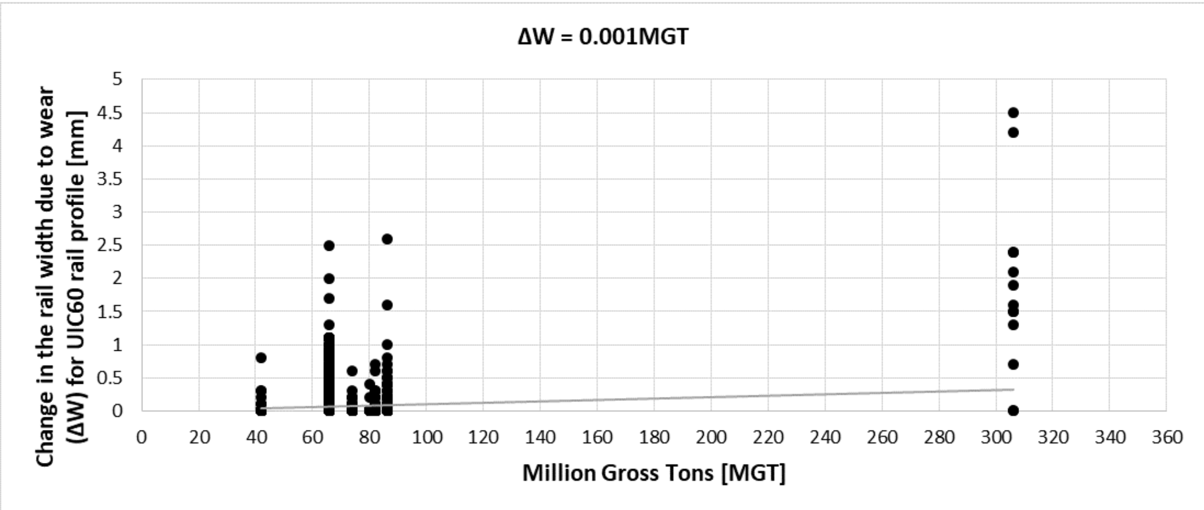


Figure 5.5 – Change in the rail width due to wear (ΔW) for UIC60 rail profile.

$$p_{W60}(MGT) = \frac{\Delta W(MGT + 8) - \Delta W(MGT)}{1} = 0.008, MGT = 0, 8, 16, 24, \dots, 344 \quad (5.5)$$

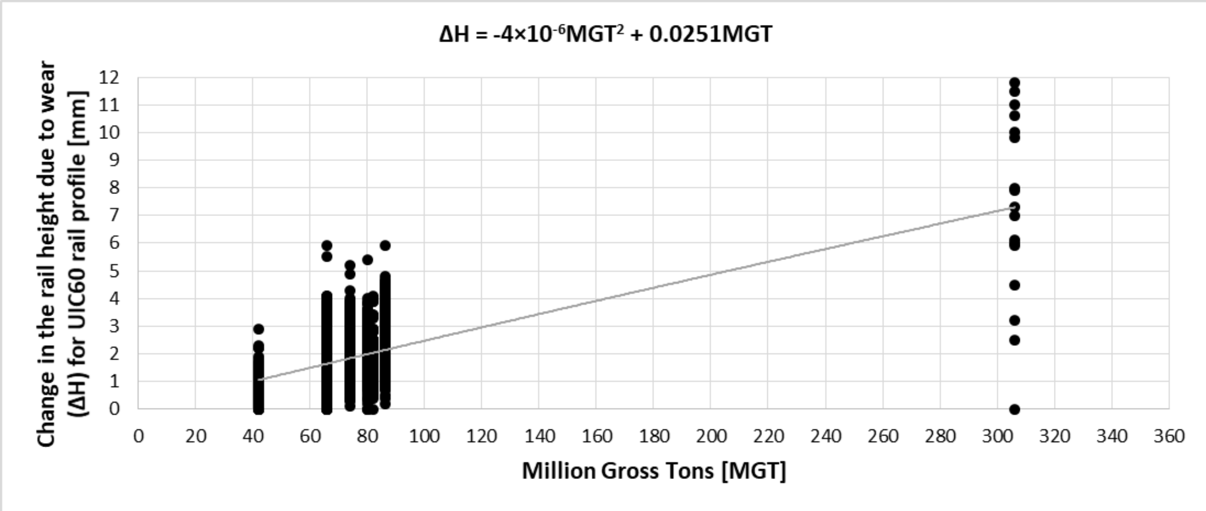


Figure 5.6 – Change in the rail height due to wear (ΔH) for UIC60 rail profile.

$$p_{H60}(MGT) = \frac{\Delta H(MGT+8) - \Delta H(MGT)}{1}, MGT = 0, 8, 16, 24, \ldots, 344 \tag{5.6}$$

5.3.2. Damage analysis

The first stage of analysis is based on data provided by the Portuguese railway infrastructure manager. This data contains information about all the defects that were detected in 2017 and 2018 inspections of “Linha do Norte” and the correspondent track kilometric points. Several defects were detected in these two years of inspections but some of these defects had already been detected in previous inspections. For this reason, it is more difficult to find out after how many years of rail lifetime they have appeared. Consequently, from these several defects detected in 2017 and 2018 inspections, it is important to filter the defects that were detected in these inspections for the first time. These filtered defects are assumed to have occurred exactly in the year of inspection.

The second stage is to analyse data from “Linha do Norte” year of renewal of each track kilometric point in order to find out after how many years of rail lifetime these defects have occurred or been detected. Dividing the number of defects detected by the respective track kilometric extension in which these defects were detected it is possible to obtain an estimate of the number of defects detected per kilometre per year as it is assumed that these defects occurred exactly in a time interval of a year. These points are plotted in Figure 5.7 and represent the defects detected per kilometre per year for each year of rail lifetime. Due to a lack of data for this particular case, it is difficult to predict the rail maximum lifetime so based on previous studies [2], it is here assumed to be 45 years.

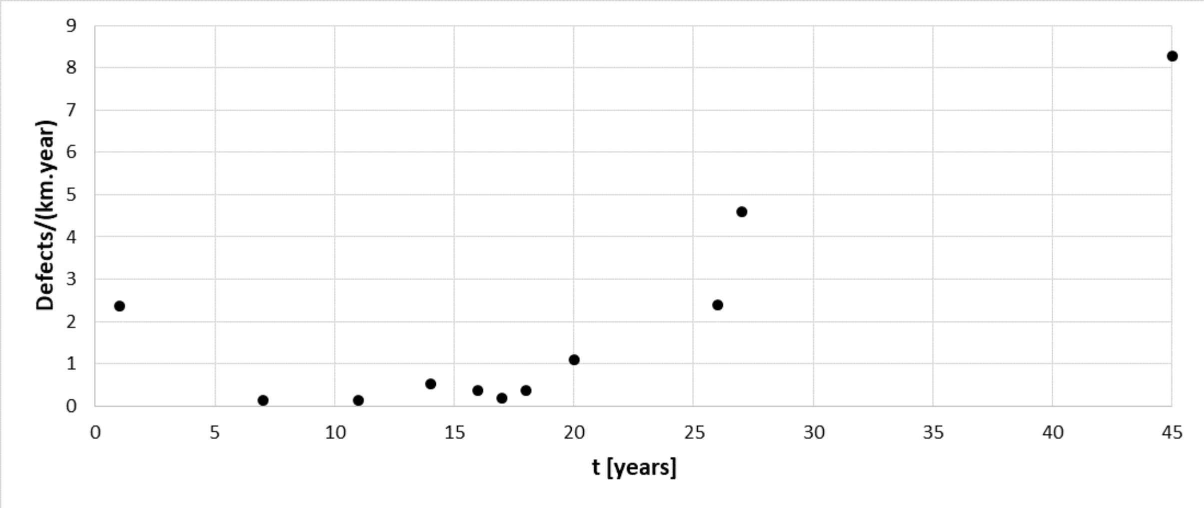


Figure 5.7 – Experimental values of the defects detected per kilometre per year for each year of rail lifetime.

The points represented in the graph above can be modelled as an additive Weibull model's Bathtub-Shaped Curve $B(t)$ proposed in [36] as a function of t, in years, which consists on the combination of two Weibull distributions and is given by:

$$B(t) = ab(at)^{b-1} + cd(ct)^{d-1}, t \geq 0 \tag{5.7}$$

The first term represents a Weibull distribution with a decreasing failure rate and the second term represents a Weibull distribution with an increasing failure rate. The parameters a, b, c and d are estimated by minimizing the nonlinear least-square errors (with the MATLAB function *slqcurvefit*). Given initial values $a = 0.05$, $b = 2$, $c = 10$ and $d = 0.2$, the solution obtained is $a = 0.0895$, $b = 3.3911$, $c = 21\,499\,000$ and $d = 0.1539$. By replacing this solution on equation (5.7), the Weibull model $B(t)$ is obtained and can be compared with the experimental values (see Figure 5.8).

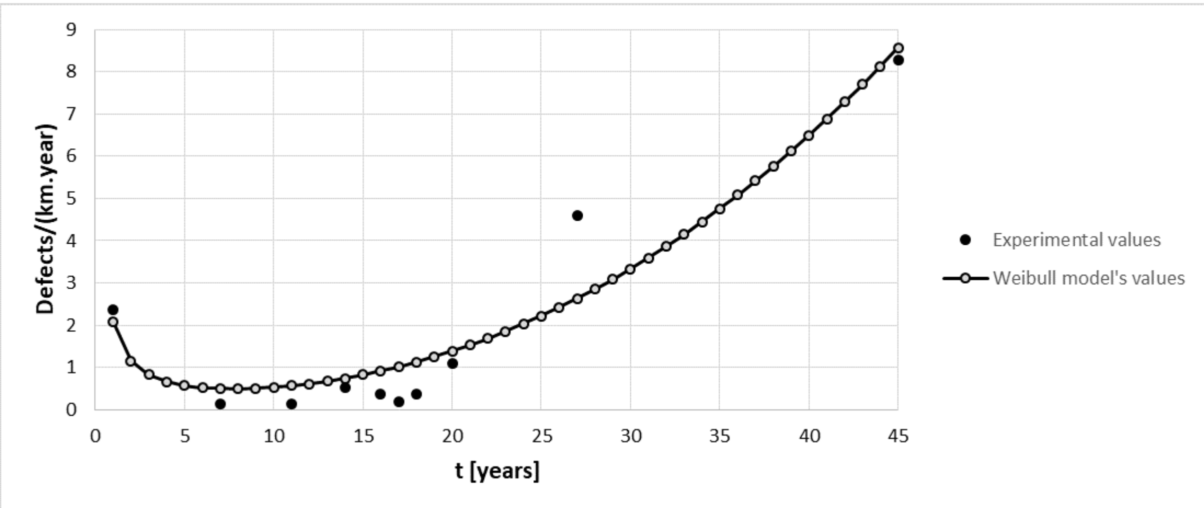


Figure 5.8 – Comparison between the experimental values shown in Figure 5.7 and the values obtained from the additive Weibull model.

Having estimated $B(t)$ for our rail damage occurrence data, the rate of defects per kilometre per year can be predicted for each of the 45 years of rail lifetime. The number of accumulated defects per kilometre $N(t)$ in defects/km for each of the 45 years of rail lifetime t are obtained from equation (5.8) and represented in Figure 5.9.

$$N(t) = \sum_{j=0}^{t} B(j)\,, t = 0,1, \dots ,45 \tag{5.8}$$

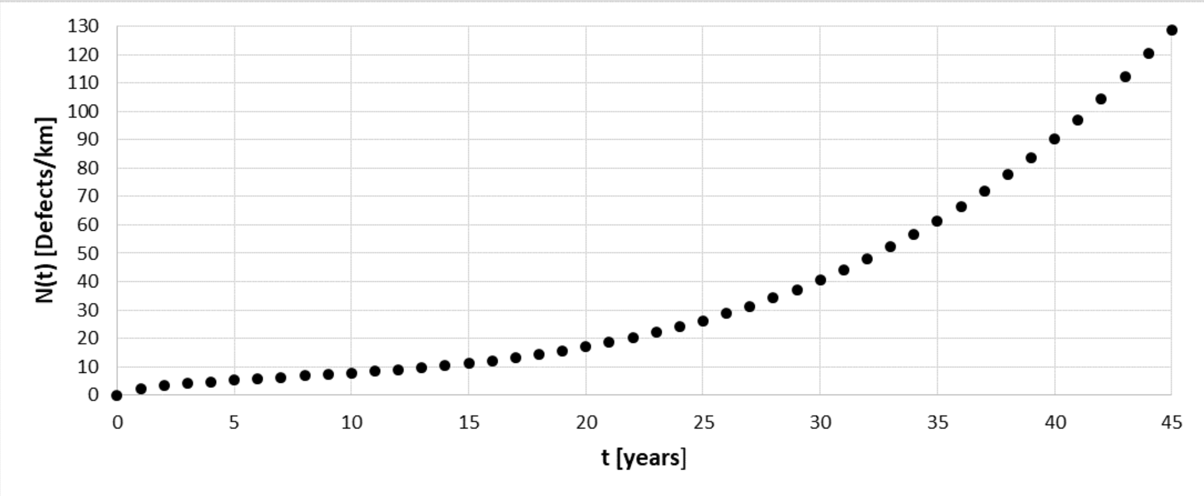


Figure 5.9 – Accumulated defects per kilometre for each of the 45 years of rail lifetime.

It is now possible to do a survival analysis considering a group of 128.85 individuals in order to obtain the reliability values for each of the 45 years of rail lifetime. An annual average traffic value of 8 MGT is assumed for “Linha do Norte”. The reliability curve obtained for this case is shown in Figure 5.10.

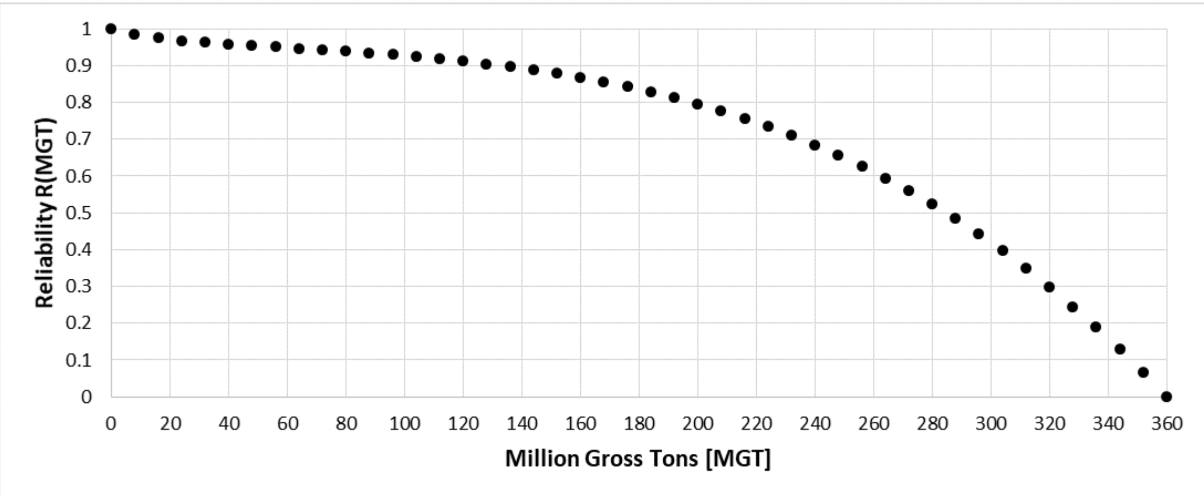


Figure 5.10 – Reliability values for each of the 45 years of rail lifetime (a maximum of 360 MGT).

The cumulative hazard rate $H(MGT)$ is obtained by replacing in expression (5.9) the reliability $R(MGT)$ values obtained previously. Since reliability $R(MGT)$ value at the end of 360 MGT is considered to be zero, the cumulative hazard rate can only be estimated for values from 0 until 352 MGT as:

$$H\ (MGT) = -\ln\big(R(MGT)\big)\,, MGT \in \{0, 8, 16, 24, \dots, 352\} \tag{5.9}$$

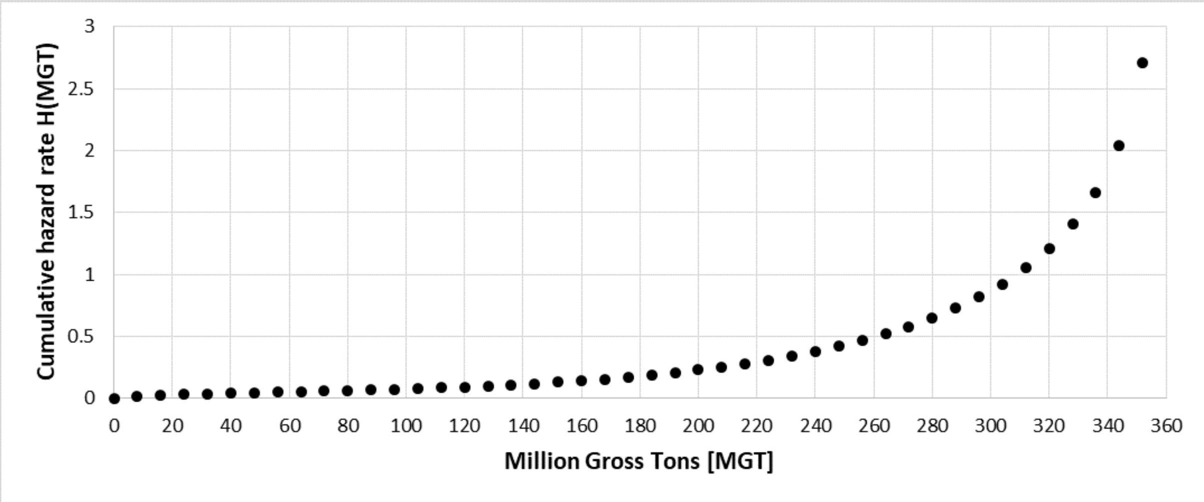


Figure 5.11 – Cumulative hazard rate.

The hazard rate associated with damage can then be estimated using expression (5.10), which is assumed to be equal to the probability of transiting to a damaged state:

$$h_{Damage}(MGT) = p_{Damage}(MGT) = H(MGT + 8) - H(MGT), \tag{5.10}$$

$$MGT \in \{0, 8, 16, 24, \dots, 344\}$$

This expression represents the probability that a rail at a certain state at epoch *n* transits to a state with damage at epoch *n+1*. These probabilities depend on the rail accumulated MGT at epoch *n*.

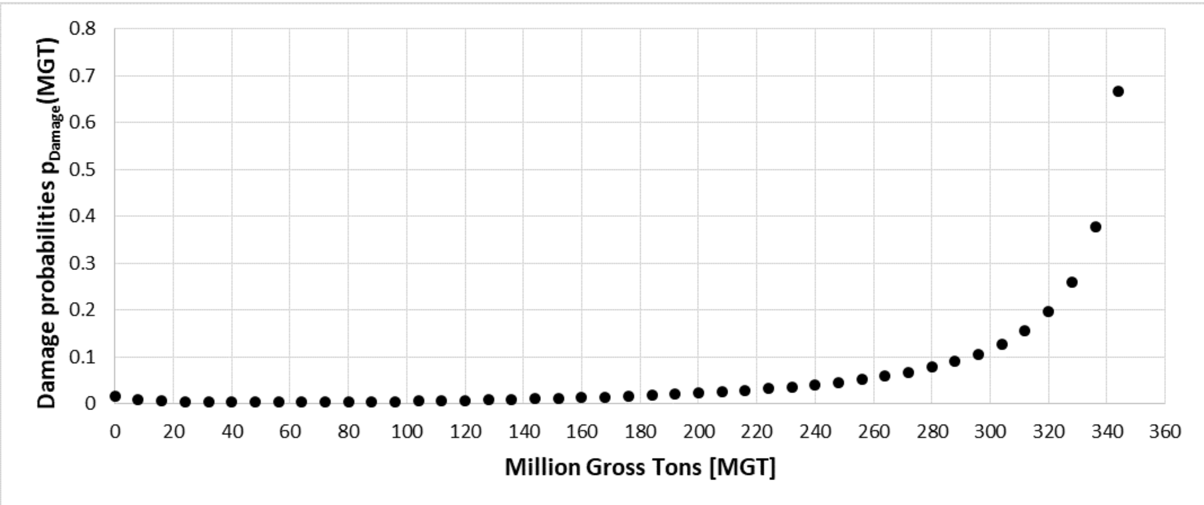


Figure 5.12 – Damage probabilities.

Although the wear analysis is based on data from “Linha de Cintura”, the number of defects detected in these two years of inspections is not representative enough to build a consistent and credible damage analysis. Therefore, unlike in the wear analysis, the damage analysis is based on data from “Linha do Norte”.

5.3.3. Markov Transition Matrices

Considering transitions in wear and in damage as independent events, the probability of their joint transitions can be computed as:

$$p(Transition\ in\ Wear\ \wedge\ Transition\ to\ Damage) \\ = p(Transition\ in\ Wear).p(Transition\ to\ Damage) \quad (5.11)$$

The transition probabilities to states without damage are written in expressions (5.3) - (5.6) and the transition probabilities to states with damage in expression (5.10). They all depend on the rail accumulated MGT at epoch *n* and are independent of the rail width (W) or height (H) states.

Regarding the “Do Nothing” action ($a = 1$) MTM probabilities for both UIC54 and UIC60 rail profiles, several assumptions are made:

- The probability of an increase in rail width (W) or height (H) states is assumed to be zero;
- The transitions to the next states are limited, which means that a two or more intervals decrease of rail width (W) or height (H) states is considered impossible, i.e. with null probability;
- The probability of a state transition only depends on the rail accumulated MGT, regardless of the rail width (W) or height (H) states;
- The transition step between each epoch is 8 MGT. Therefore, when the rail transits to a state without damage, it transits to a state with 8 more MGT, unless it is in a state with damage or in a state with 352 accumulated MGT;
- The state transition to a damaged state assumes that the rail maintains the same width (W) and height (H) states.

Bearing in mind these five assumptions, four possible cases concerning state transition probabilities from epoch *n* to epoch *n+1* are worth discussing by considering the rail is in:

1) A state with 352 accumulated MGT at epoch *n;*
2) A damaged state at epoch *n;*
3) A scrap state (see definition in section 5.2) with accumulated MGT value different from 352 at epoch *n*;
4) A non-scrap state with accumulated MGT value different from 352 at epoch *n*.

These four cases are illustrated as follows:

1) For this case it is assumed that it transits to a damaged state at epoch *n+1*. Thus the rail maintains the same width (W) and height (H) states at epoch *n+1*.

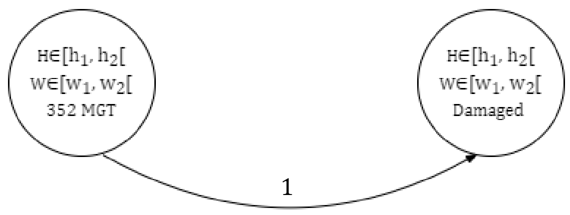


Figure 5.13 – Possible transitions for a state with 352 accumulated MGT for the “Do Nothing” action ($a = 1$).

2) For a damaged state it is certain that the rail remains in the same state at epoch *n+1*.

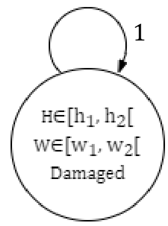


Figure 5.14 - Possible transitions for a damaged state for the “Do Nothing” action ($a = 1$).

3) For a rail scrap state with accumulated MGT value different from 352, two different transitions to epoch *n+1* are possible.

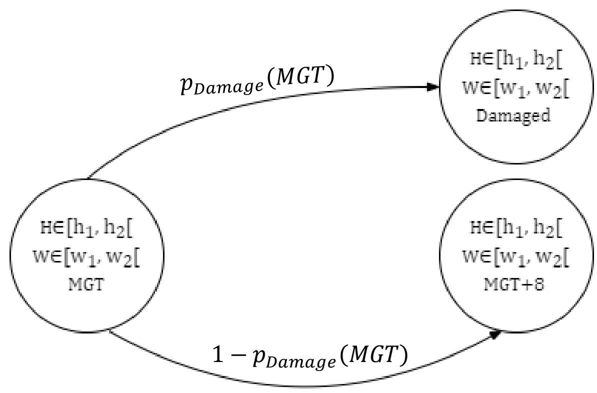


Figure 5.15 - Possible transitions for a scrap state with accumulated MGT value different from 352 for the “Do Nothing” action ($a = 1$).

4) Considering that the rail is in a non-scrap state with accumulated MGT value different from 352, five different transitions to epoch *n+1* are possible.

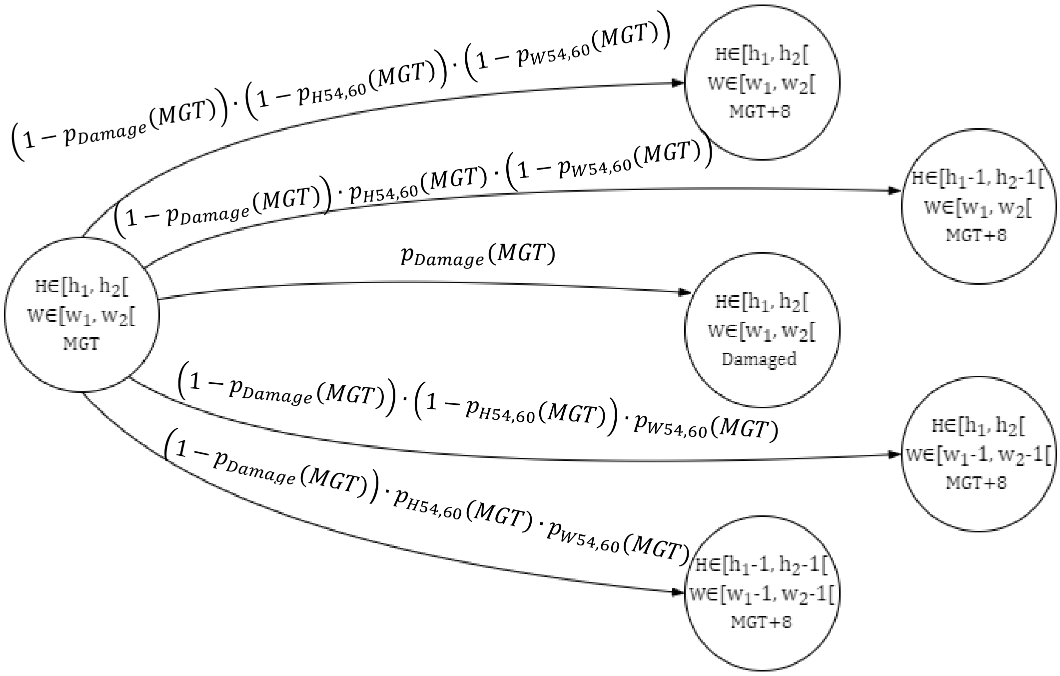


Figure 5.16 – Possible transitions for a non-scrap state with accumulated MGT value different from 352 for the "Do Nothing" action ($a = 1$).

For the two UIC54 and UIC60 rail profiles, the correspondent MTMs P_1^{54} (8 372 × 8 372) and P_1^{60} (11 776 × 11 776) respectively, for the "Do Nothing" action ($a = 1$) are divided into two sub-matrices: P_{Wear} and P_{Damage} (see Appendices B1 and B2) and assume the following form:

$$
\begin{array}{c}
\begin{array}{ccccccc}
0\,MGT & 8\,MGT & 16\,MGT & \ldots & 344\,MGT & 352\,MGT & Damage\ states \\
\downarrow & \downarrow & \downarrow & & \downarrow & \downarrow & \downarrow
\end{array} \\
P_1^{\substack{54\\60}} = \begin{bmatrix}
0 & P_{Wear}(0) & 0 & \cdots & 0 & 0 & P_{Damage}(0) \\
0 & 0 & P_{Wear}(8) & \cdots & 0 & 0 & P_{Damage}(8) \\
0 & 0 & 0 & \cdots & 0 & 0 & P_{Damage}(16) \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & P_{Wear}(344) & P_{Damage}(344) \\
0 & 0 & 0 & 0 & 0 & 0 & I \\
0 & 0 & 0 & 0 & 0 & 0 & I
\end{bmatrix}
\begin{array}{l}
\leftarrow 0\,MGT \\
\leftarrow 8\,MGT \\
\leftarrow 16\,MGT \\
\quad \vdots \\
\leftarrow 344\,MGT \\
\leftarrow 352\,MGT \\
\leftarrow Damage\ states
\end{array}
\end{array}
\quad (5.12)
$$

5.4. MTM for the “Renewal” action $(\boldsymbol{a} = \mathbf{2})$

The “Renewal” action $(a = 2)$ consists in assuming that the rail is unable to continue in service and replace it, regardless of the state it is. Therefore, for every state the rail is at epoch *n,* when a “Renewal” action occurs, it is certain that it transits to the initial state s_1 at epoch *n+1*. The MTMs for both UIC54 (P_2^{54}) and UIC60 (P_2^{60}) rail profiles are represented below.

- **UIC54 rail profile**

$$P_2^{54} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix} \quad (8\,372 \times 8\,372) \tag{5.13}$$

- **UIC60 rail profile**

$$P_2^{60} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix} \quad (11\,776 \times 11\,776) \tag{5.14}$$

5.5. MTM for the “Grinding” action $(\boldsymbol{a} = \mathbf{3})$

The “Grinding” action $(a = 3)$ consists in removing small or more severe surface defects on the rail head through the rotational movement of grinding stones. As mentioned previously, grinding can be of three types: initial, preventive or corrective. For this particular practical case, only preventive grinding (for the undamaged states) and corrective grinding (for the damaged states) are considered. Regarding the “Grinding” action $(a = 3)$ MTM probabilities for both UIC54 and UIC60 rail profiles, four assumptions are made:

- Only rail height (H) is affected, which means that the rail remains in the same width (W) state and can only decrease intervals in height (H) state in each transition;
- The rail transits to a state with 0 accumulated MGT since it is considered to be completely repaired, and a new damage cycle is reinitiated;
- Unlike in the “Do Nothing” action case, the transition probabilities for preventive and corrective grinding are independent of the rail accumulated MGT;
- As it is not possible to grind a rail beyond the minimum interval in height (H) state, when the rail reaches this condition and the probabilities obtained indicate height losses that go beyond that minimum interval in height (H) state, then the probabilities of the remaining transitions are summed up, becoming the probability value to stay at the state with minimum interval in height (H).

5.5.1. Preventive grinding

For the preventive grinding, the transitions to the next states are limited, which means that a two or more intervals decrease of rail height (H) state is considered impossible. Denoting p_{pg} as the probability of a one interval decrease in rail height (H) state, the possible transitions are represented below for a rail height (H) state above the minimum interval.

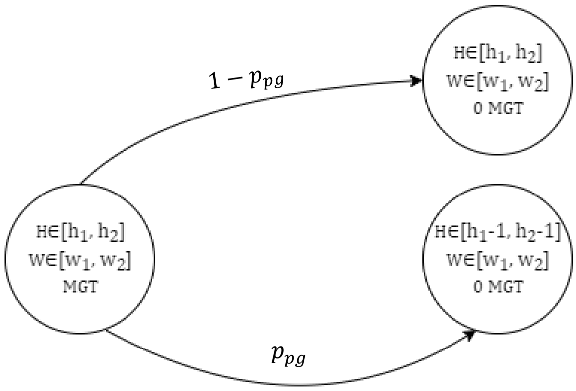


Figure 5.17 - Possible transitions for a state with height (H) above the minimum interval for the preventive "Grinding" action ($a = 3$).

Based on the information provided by the Portuguese railway infrastructure manager, the average rail wear μ_{pg} associated with a preventive grinding is around 0.3 mm. Considering a wear of 0 mm for the case in which the rail remains in the same height (H) state, and 1 mm of wear for the one interval decrease in height (H) state, the transition probability p_{pg} can then be obtained as follows:

$$\mu_{pg} = 0.3 \Leftrightarrow p_{pg} \cdot (1) + (1 - p_{pg}) \cdot (0) = 0.3 \Leftrightarrow p_{pg} = 0.3 \tag{5.15}$$

5.5.2. Corrective grinding

For the corrective grinding, the number of occurrences of four different crack lengths observed in a railway track is obtained from [37]. Since there is no statistical data available of the change in the rail height as a result of corrective grinding operation, the corrective grinding depth is inferred from this sample of crack lengths and is assumed to follow the same statistical distribution of these crack lengths (see Table 5.5).

Table 5.5 – Number of occurrences of four different crack lengths.

Occurrence number (i)	**Crack length (x_i)**	**Number of occurrences (F_i)**	**Relative frequency (f_i)**
1	4 mm	7	7/24
2	5 mm	8	8/24
3	6 mm	8	8/24

4	9 mm	1	1/24
		$\sum F_i = 24$	$\sum f_i = 1$

A normal distribution is used to model the change in the rail height (H) as a result of a corrective grinding operation and is denoted by $X \sim N(\mu, \sigma^2)$. The parameters μ and σ are estimated below:

$$\hat{\mu} = 5.208333 \text{ mm} \tag{5.16}$$

$$\hat{\sigma} = 1.141287 \text{ mm} \tag{5.17}$$

The MTM probabilities for the corrective grinding p_{cg} are shown in Table 5.6. Bearing in mind the two estimated parameters of the normal distribution, the values of the change in the rail height (H) as a result of a corrective grinding operation are assumed to be between 0 and 10 mm. However, it is expectable that a small fraction of the probability density function is below 0 mm or above 10 mm. Considering this, a truncated normal distribution is used to bound the modelled variable within the specified range of values.

Table 5.6 – MTM probabilities for the corrective grinding.

Probability number (*j*)	$\boldsymbol{x_{min}}$	$\boldsymbol{x_{max}}$	$\boldsymbol{p_{cg\,j}(x_{min} \leq X < x_{max})}$
1	0 mm	1 mm	0.000111
2	1 mm	2 mm	0.002355
3	2 mm	3 mm	0.02403
4	3 mm	4 mm	0.118361
5	4 mm	5 mm	0.282725
6	5 mm	6 mm	0.328479
7	6 mm	7 mm	0.185727
8	7 mm	8 mm	0.051003
9	8 mm	9 mm	0.006775
10	9 mm	10 mm	0.000434

5.5.3. Markov Transition Matrices

For the two UIC54 and UIC60 rail profiles, the corresponding MTMs $P_3^{54}\,(8\,372 \times 8\,372)$ and $P_3^{60}\,(11\,776 \times 11\,776)$ for the "Grinding" action ($a = 3$) are divided into two sub-matrices: P_{PG} and P_{CG} for preventive grinding, and corrective grinding, respectively, and assume the following form:

$$
\begin{array}{c}
\begin{array}{ccccc} 0\,MGT & 8\,MGT & \cdots & 352\,MGT & Damage\ states \\ \downarrow & \downarrow & & \downarrow & \downarrow \end{array} \\
P_3^{\substack{54\\60}} = \begin{bmatrix} P_{PG} & 0 & \cdots & 0 & 0 \\ P_{PG} & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ P_{PG} & 0 & 0 & 0 & 0 \\ P_{CG} & 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} \leftarrow 0\,MGT \\ \leftarrow 8\,MGT \\ \leftarrow \vdots \\ \leftarrow 352\,MGT \\ \leftarrow Damage\ states \end{array}
\end{array}
\tag{5.18}
$$

For further details, on the two sub-matrices P_{PG} and P_{CG}, the reader is remitted to Appendices C1 and C2.

5.6. Rewards/cost function

The MATLAB toolbox function chosen to solve this problem uses a reward maximization to derive an optimal policy which maximizes the total rewards earned over an infinite horizon. Therefore, the values used to represent the costs associated with each maintenance action must be negative. To derive the rewards/cost function, a reward vector for each action ($a = 1,2,3$) must be specified as written in equation (4.14).

5.6.1. "Do Nothing" action $(\boldsymbol{a = 1})$

The "Do Nothing" action ($a = 1$) does not hold any operational cost. However, it is important to guarantee that when the rail reaches scrap states, states with 352 accumulated MGT or damaged states, other option different from "Do Nothing" action must be chosen. This is achieved by giving to these critical states cost values larger than the ones used in the other two possible actions. Thus, a penalty of -200 thousand monetary units/km is assigned to these critical states.

- **UIC54 rail profile**

The reward vector for the UIC54 rail profile, $q^{1\,54}(8\,372 \times 1)$ (see equation (5.19)) is divided into two sub-vectors, $q_{\alpha}^{1\,54}(182 \times 1)$ and $q_{\beta}^{1\,54}(182 \times 1)$ (see equations (5.20) and (5.21), respectively).

$$q^{1\,54} = \begin{bmatrix} q_{\alpha}^{1\,54} \\ q_{\alpha}^{1\,54} \\ \vdots \\ q_{\alpha}^{1\,54} \\ q_{\beta}^{1\,54} \\ q_{\beta}^{1\,54} \end{bmatrix} \begin{matrix} \leftarrow & 0\ MGT \\ \leftarrow & 8\ MGT \\ \leftarrow & \vdots \\ \leftarrow & 344\ MGT \\ \leftarrow & 352\ MGT \\ \leftarrow & Damage\ states \end{matrix} \tag{5.19}$$

$$q_{\alpha\,i}^{1\,54} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ q_{\alpha\,13}^{1\,54} \\ 0 \\ \vdots \\ 0 \\ q_{\alpha\,26}^{1\,54} \\ 0 \\ \vdots \\ 0 \\ q_{\alpha\,169}^{1\,54} \\ q_{\alpha\,170}^{1\,54} \\ \vdots \\ q_{\alpha\,182}^{1\,54} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ -200\,000 \\ 0 \\ \vdots \\ 0 \\ -200\,000 \\ 0 \\ \vdots \\ 0 \\ -200\,000 \\ -200\,000 \\ \vdots \\ -200\,000 \end{bmatrix} \tag{5.20}$$

$$q_{\beta\,i}^{1\,54} = \begin{bmatrix} q_{\beta\,1}^{1\,54} \\ \vdots \\ q_{\beta\,182}^{1\,54} \end{bmatrix} = \begin{bmatrix} -200\,000 \\ \vdots \\ -200\,000 \end{bmatrix} \tag{5.21}$$

- **UIC60 rail profile**

The reward vector for the UIC60 rail profile, $q^{1\,60}(11\,776 \times 1)$ (see equation (5.22)) is divided into two sub-vectors, $q_{\alpha}^{1\,60}(256 \times 1)$ and $q_{\beta}^{1\,60}(256 \times 1)$ (see equations (5.23) and (5.24), respectively).

$$q^{1\,60} = \begin{bmatrix} q_{\alpha}^{1\,60} \\ q_{\alpha}^{1\,60} \\ \vdots \\ q_{\alpha}^{1\,60} \\ q_{\beta}^{1\,60} \\ q_{\beta}^{1\,60} \end{bmatrix} \begin{matrix} \leftarrow & 0\ MGT \\ \leftarrow & 8\ MGT \\ \leftarrow & \vdots \\ \leftarrow & 344\ MGT \\ \leftarrow & 352\ MGT \\ \leftarrow & Damage\ states \end{matrix} \tag{5.22}$$

$$q_{\alpha\, i}^{1\,60} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ q_{\alpha\, 16}^{1\,60} \\ 0 \\ \vdots \\ 0 \\ q_{\alpha\, 32}^{1\,60} \\ 0 \\ \vdots \\ 0 \\ q_{\alpha\, 240}^{1\,60} \\ q_{\alpha\, 241}^{1\,60} \\ \vdots \\ q_{\alpha\, 256}^{1\,54} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ -200\,000 \\ 0 \\ \vdots \\ 0 \\ -200\,000 \\ 0 \\ \vdots \\ 0 \\ -200\,000 \\ -200\,000 \\ \vdots \\ -200\,000 \end{bmatrix} \tag{5.23}$$

$$q_{\beta\, i}^{1\,60} = \begin{bmatrix} q_{\beta\, 1}^{1\,60} \\ \vdots \\ q_{\beta\, 256}^{1\,60} \end{bmatrix} = \begin{bmatrix} -200\,000 \\ \vdots \\ -200\,000 \end{bmatrix} \tag{5.24}$$

5.6.2. "Renewal" action $(a = 2)$

It was considered for the "Renewal" action ($a = 2$) a cost value of -67.554 thousand monetary units/km, regardless of the state the rail is. The reward vectors for the UIC54 and UIC60 rail profiles, $q^{2\,54}(8\,372 \times 1)$ and $q^{2\,60}(11\,776 \times 1)$ respectively, are represented in equations (5.25) and (5.26).

- **UIC54 rail profile**

$$q^{2\,54} = \begin{bmatrix} q_1^{2\,54} \\ \vdots \\ q_{8\,372}^{2\,54} \end{bmatrix} = \begin{bmatrix} -67\,554 \\ \vdots \\ -67\,554 \end{bmatrix} \tag{5.25}$$

- **UIC60 rail profile**

$$q^{2\,60} = \begin{bmatrix} q_1^{2\,60} \\ \vdots \\ q_{11\,776}^{2\,60} \end{bmatrix} = \begin{bmatrix} -67\,554 \\ \vdots \\ -67\,554 \end{bmatrix} \tag{5.26}$$

5.6.3. "Grinding" action $(a = 3)$

It was chosen a value of -22.630 thousand monetary units/km for the "Grinding" action ($a = 3$). However, when the rail reaches a scrap state, a "Renewal" action is needed rather than a "Grinding" action. Thus, similar to the case of the "Do Nothing" action, a penalty of -200 thousand monetary units/km is assigned to this critical state.

- **UIC54 rail profile**

The reward vector for the UIC54 rail profile, $q^{3\ 54}(8\ 372 \times 1)$ (see equation (5.27)) is divided into a sub-vector, $q_{\alpha}^{3\ 54}(182 \times 1)$ (see equation (5.28)).

$$q^{3\ 54} = \begin{bmatrix} q_{\alpha}^{3\ 54} \\ q_{\alpha}^{3\ 54} \\ \vdots \\ q_{\alpha}^{3\ 54} \\ q_{\alpha}^{3\ 54} \\ q_{\alpha}^{3\ 54} \end{bmatrix} \begin{matrix} \leftarrow & 0\ MGT \\ \leftarrow & 8\ MGT \\ \leftarrow & \vdots \\ \leftarrow & 344\ MGT \\ \leftarrow & 352\ MGT \\ \leftarrow & Damage\ states \end{matrix} \tag{5.27}$$

$$q_{\alpha\ i}^{3\ 54} = \begin{bmatrix} -22\ 630 \\ \vdots \\ -22\ 630 \\ q_{\alpha\ 13}^{3\ 54} \\ -22\ 630 \\ \vdots \\ -22\ 630 \\ q_{\alpha\ 26}^{3\ 54} \\ -22\ 630 \\ \vdots \\ -22\ 630 \\ q_{\alpha\ 169}^{3\ 54} \\ q_{\alpha\ 170}^{3\ 54} \\ \vdots \\ q_{\alpha\ 182}^{3\ 54} \end{bmatrix} = \begin{bmatrix} -22\ 630 \\ \vdots \\ -22\ 630 \\ -200\ 000 \\ -22\ 630 \\ \vdots \\ -22\ 630 \\ -200\ 000 \\ -22\ 630 \\ \vdots \\ -22\ 630 \\ -200\ 000 \\ -200\ 000 \\ \vdots \\ -200\ 000 \end{bmatrix} \tag{5.28}$$

- **UIC60 rail profile**

The reward vector for the UIC60 rail profile, $q^{3\ 60}(11\ 776 \times 1)$ (see equation (5.29)) is divided into a sub-vector, $q_{\alpha}^{3\ 60}(256 \times 1)$ (see equation (5.30)).

$$q^{3\ 60} = \begin{bmatrix} q_{\alpha}^{3\ 60} \\ q_{\alpha}^{3\ 60} \\ \vdots \\ q_{\alpha}^{3\ 60} \\ q_{\alpha}^{3\ 60} \\ q_{\alpha}^{3\ 60} \end{bmatrix} \begin{matrix} \leftarrow & 0\ MGT \\ \leftarrow & 8\ MGT \\ \leftarrow & \vdots \\ \leftarrow & 344\ MGT \\ \leftarrow & 352\ MGT \\ \leftarrow & Damage\ states \end{matrix} \tag{5.29}$$

$$q_{\alpha\,i}^{3\,60} = \begin{bmatrix} -22\,630 \\ \vdots \\ -22\,630 \\ q_{\alpha\,16}^{3\,60} \\ -22\,630 \\ \vdots \\ -22\,630 \\ q_{\alpha\,32}^{3\,60} \\ -22\,630 \\ \vdots \\ -22\,630 \\ q_{\alpha\,240}^{3\,60} \\ q_{\alpha\,241}^{3\,60} \\ \vdots \\ q_{\alpha\,256}^{3\,60} \end{bmatrix} = \begin{bmatrix} -22\,630 \\ \vdots \\ -22\,630 \\ -200\,000 \\ -22\,630 \\ \vdots \\ -22\,630 \\ -200\,000 \\ -22\,630 \\ \vdots \\ -22\,630 \\ -200\,000 \\ -200\,000 \\ \vdots \\ -200\,000 \end{bmatrix} \tag{5.30}$$

5.7. Optimal policy map

As mentioned previously in this chapter, the MATLAB toolbox function chosen to solve this problem uses a reward maximization to derive an optimal policy, which maximizes the total rewards earned over an infinite horizon. Therefore, a graphical representation of the decision map is obtained for UIC54 and UIC60 rail profiles for rail width (W) and height (H) states with the evolution of the accumulated Million Gross Tons (MGT). This representation is provided for the states with damage and without damage and can serve as a guideline for condition-based maintenance carried out by railway infrastructure managers. All the possible states of the rail are represented in grid cells whose colours represent the best decision for that same state. Each vertical block represents a height (H) state and all the possible width (W) states are represented for each height state (H), from top to bottom, in descending order. The decision maps for UIC54 and UIC60 rail profiles are represented, respectively, in Figures 5.18 and 5.19.

- **UIC54 rail profile**

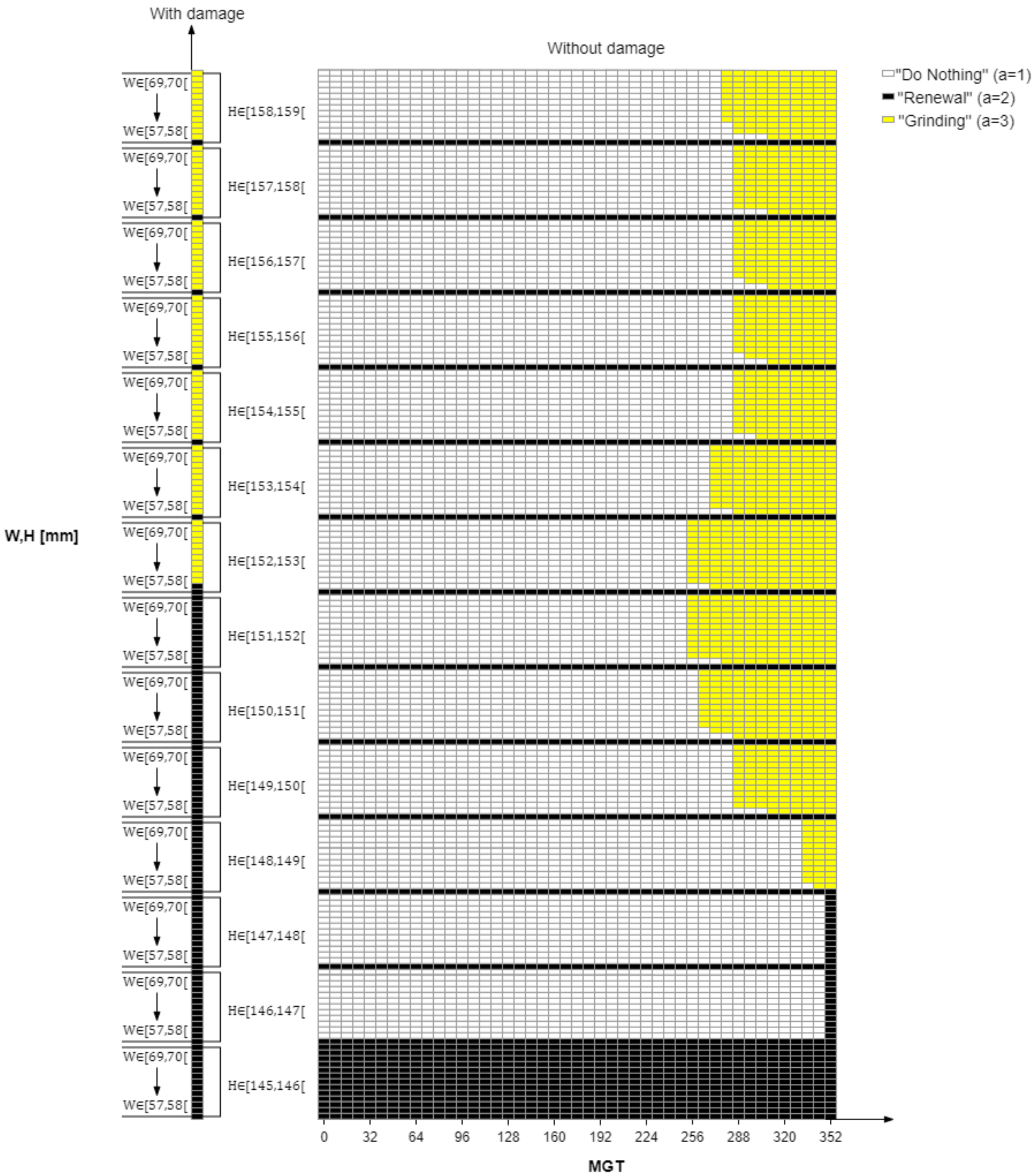


Figure 5.18 – Decision map for UIC54 rail profile.

- **UIC60 rail profile**

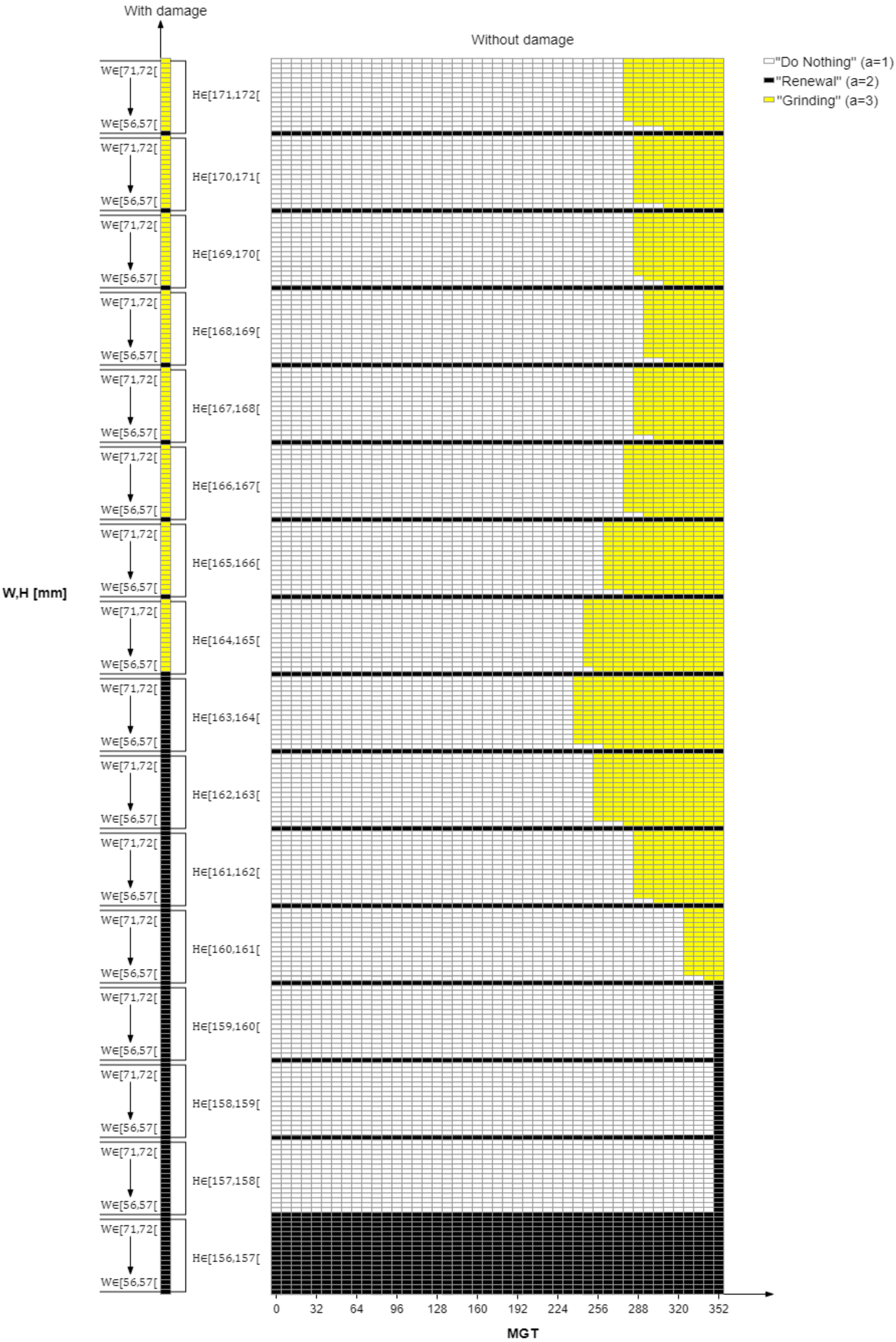


Figure 5.19 – Decision map for UIC60 rail profile.

By analysing the two decision maps obtained, one can see that:

a) For damaged rails, “Renewal” or “Grinding” actions are mandatory. A “Renewal” action is assigned to scrap states (i.e. states in which width (W) and/or height (H) states reach their minimum interval) or the last height (H) states, for all the width (W) states. For the case of the UIC54 rail profile, a “Renewal” action must be carried out for the last 7 height states (H), from 145 mm to 152 mm, for all the 13 width (W) states and for all the remaining states of minimum width (W). For the case of UIC60 rail profile, a “Renewal” action must be performed for the last 8 height states (H), from 156 mm to 164 mm, for all the 16 width (W) states and for all the remaining states of minimum width (W).
b) For undamaged rails, recommended actions change with the accumulated MGT, besides width (W) and height (H) states. “A Renewal” action is mandatory for scrap states as in the case of the damaged rails. However, for states with the maximum accumulated MGT allowable (352 MGT), “Renewal” actions are also mandatory for the last height (H) states, for all the width (W) states. For the case of UIC54 rail profile, a “Renewal” action is mandatory for the last 3 height (H) states, from 145 mm to 148 mm, for all the 13 width (W) states. For the case of UIC60 rail profile, a “Renewal” action is mandatory for the last 4 height (H) states, from 156 mm to 160 mm, for all the 16 width (W) states.

For undamaged rails, the “Grinding” action must be analysed for the two profiles:

a) For the case of UIC54 rail profile, the lowest value of the accumulated MGT variable for which a preventive grinding is recommended is 256 MGT and it occurs for height (H) states between 151 mm and 153 mm. As height (H) states decrease below 151 mm, preventive grinding actions should be performed for higher values of the accumulated MGT. When the rail reaches a height (H) value of 148 mm or below, preventive grinding actions are not recommended anymore. On the other hand, as height (H) states decrease from 159 mm until 153 mm, two tendencies are verified.
b) For the case of UIC60 rail profile, a preventive grinding is recommended until accumulated MGT values of 240 and it occurs for height (H) states between 163 mm and 164 mm. Preventive grinding actions are recommended later as height states decrease below 163 mm. Preventive grinding actions are not recommended anymore for height (H) values of 160 mm or below, so the best strategy is to not perform preventive grinding at all and let the rail degrade until a scrap state or until 352 MGT, in which a “Renewal” action must be carried out. On the other hand, as height (H) states decrease from 172 mm until 164 mm, two tendencies are verified as in the case of UIC54 rail profile.

6. Conclusions and Further Research

This final chapter presents the main conclusions of the present dissertation, identifies some limitations and suggests future paths for further research.

6.1. Conclusions

Railway maintenance is a procedure where railway infrastructures undergo a maintenance schedule on a regular basis in order to prevent the occurrence of failures and to maintain railway components within the tolerance ranges specified in the standards. A condition-based maintenance plan can be obtained using a Markov Decision Process (MDP) approach, optimizing rail maintenance procedure from a life-cycle and an economic point of view.

An MDP approach was applied to a practical case of Portuguese railway lines to model rail degradation in terms of rail height (H), width (W), accumulated Million Gross Tons (MGT) and the occurrence of damage. According to these four indicators, an optimal maintenance plan is obtained for UIC54 and UIC60 rail profiles. The state space was divided into 8 372 and 11 776 states, respectively for UIC54 and UIC60 rail profiles. A set of three possible actions were defined: i) “Do Nothing”, ii) “Renewal” and iii) “Grinding”. An optimal decision policy was derived using linear programming and a decision map was provided with the aim of supporting the decision-maker to take the best maintenance decision for each possible rail condition state.

From a detailed analysis of the two decision maps obtained, one can conclude that for damaged rails, only “Renewal” or “Grinding” actions must be performed. “Renewal” actions are mandatory for scrap states since a “Grinding” action would wear the rail beyond acceptable values. For UIC54 rail profile, the “Renewal” action is assigned to rail heights (H) until 7 mm above the minimum interval (145 mm), for all the 13 width (W) states. On the other hand, for UIC60 rail profile, a “Renewal” action is mandatory for rail heights (H) until 8 mm above the minimum interval (156 mm), for all the 16 width (W) states. For undamaged rails, “Renewal” actions are also mandatory for scrap states as one might expect. Comparing the two rail profiles for undamaged states with 352 MGT, for the case of UIC54 rail profile, a “Renewal” action must be carried out for height (H) states until 3 mm above the minimum interval in height (H) state, for all the 13 width (W) states, whereas for the case of UIC60 rail profile, a “Renewal” action must be carried out for height (H) states until 4 mm above the minimum interval in height (H) state, for all the 16 width (W) states. In a nutshell, UIC60 rail profile requires that a “Renewal” action must be performed for height (H) values more above the minimum interval than UIC54 rail profile, for both damaged rails and undamaged rails with 352 MGT.

Comparing the two rail profiles for the “Grinding” action performed for undamaged rails, it can be derived that for UIC54 rail profile it is required for rails with accumulated MGT between 256 and 352 and height (H) states between 148 mm and 159 mm, except in rail scrap states. For the case of UIC60 rail profile, a “Grinding” action would be advisable for rails with accumulated MGT between 240 and 352 and height

(H) states between 160 mm and 172 mm, except in rail scrap states. As mentioned in section 5.7, the lowest value of the accumulated MGT variable for which a preventive grinding is recommended is 256 MGT for UIC54 rail profile and 240 MGT for UIC60 rail profile. In general, for UIC54 rail profile, preventive grinding actions should be performed later (i.e. for higher values of the rail accumulated MGT). As height (H) states decrease below 151 mm for UIC54 rail profile or below 163 mm for UIC60 rail profile, the preventive grinding would be advisable later. The decreasing slope formed in the maps resulting from the transitions between the “Do Nothing” and “Grinding” actions is approximately the same for the two rail profiles.

Overall, these optimal policies require railway infrastructure companies to have a tight control over their assets, in particular railway lines, in order to constantly monitor the actual condition of the rails and perform the adequate maintenance actions according to these optimal policies.

6.2. Limitations

In the present dissertation, several limitations can be identified. First of all, the MDP approach is a random (or stochastic) process that requires defining transition probabilities, which are calibrated based on data from past inspections/samples. Random factors such as the month of inspection or meteorological conditions under which these inspections were carried out may influence the values obtained.

Relatively to the Markov Transition Matrices (MTM) for the “Grinding” action, the probabilities of the corrective grinding were obtained based on the number of occurrences of four different crack lengths observed in the rail component. It is inferred that when a defect occurs, the corrective grinding depth is equal to the crack length, which is a rough assumption. Although the state space was sufficiently large to describe the different width (W), height (H), accumulated MGT and damage occurrence states, it did not control the evolution of track geometric parameters such as gauge, cross level, twist, alignment and longitudinal level, which would be desirable to model railway track as a whole system.

Moreover, only accumulated MGT and rail profile (P) are considered to influence rail degradation in the estimation of the MTM probabilities. Several explanatory variables such as rail relative position (RP) or rail curvature (C) considered in section 5.1 play also a key role in rail degradation process, though they have not been considered in the estimation of the MTM probabilities. As mentioned previously in this dissertation, this is mainly due to the need to make the MDP formulation simpler.

Finally, one of the main limitations regarding the methodology used is that the Markov chain model does not account for the presence of measurement errors inherent to inspection activities. Thus, data may have been collected with different measuring precisions over time.

6.3. Further Research

Bearing in mind all the limitations pointed out previously, several steps for further research are suggested in order to overcome these obstacles. For the present dissertation, only information about the number of occurrences of different crack lengths was obtained. However, for further research, data from rail grinding operations carried out in the track section analysed should be collected in order to improve the accuracy in the MTM estimation for the “Grinding” action.

Moreover, the explanatory variables considered in section 5.1 should be integrated in a next MDP approach. For instance, rail data from different track curvatures should be analysed separately and a decision map must be provided for each of those track sections in order to improve the feasibility of the results obtained. It would also be recommended to include track geometric parameters in the form of a Track Quality Indicator as a variable to define the state space. Bearing in mind all these variables, an improved optimal maintenance plan would be obtained.

Finally, the presence of measurement errors can be accounted by the usage of some extensions of the MDP such as the Latent MDP or the Partially Observed MDP. These optimization processes deal with the presence of measurement uncertainties associated with inspection activities.

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Appendix

A1 Definition of MDP state space for UIC54 rail profile

For UIC54 rail profile maintenance plan, the state space with the rail width (W) is divided into 13 different states.

$s = \{s_1 = 1$	$s_2 = 2$	...	$s_{13} = 13\}$
$W \in [69,70[$	$W \in [68,69[$		$W \in [57,58[$

Rail height (H) state space is divided into 14 different states. Therefore, the state space with the rail width (W) and height (H) is divided into 182 states, as shown below.

$s = \{s_1 = 1$	$s_2 = 2$	...	$s_{13} = 13$
$H \in [158,159[$	$H \in [158,159[$		$H \in [158,159[$
$W \in [69,70[$	$W \in [68,69[$		$W \in [57,58[$
$s_{14} = 14$	$s_{15} = 15$	...	$s_{26} = 26$
$H \in [157,158[$	$H \in [157,158[$		$H \in [157,158[$
$W \in [69,70[$	$W \in [68,69[$		$W \in [57,58[$
$\vdots$	$\vdots$		$\vdots$
$s_{170} = 170$	$s_{171} = 171$	...	$s_{182} = 182\}$
$H \in [145,146[$	$H \in [145,146[$		$H \in [145,146[$
$W \in [69,70[$	$W \in [68,69[$		$W \in [57,58[$

Besides rail height (H) and width (W), another important variable that must be considered to define the state space is the accumulated MGT, which is divided into 45 states, from 0 MGT to 352 MGT. Therefore, the state space with the rail width (W), height (H) and accumulated MGT is divided into 8 190 states, as shown below.

$s = \{s_1 = 1$	$s_2 = 2$	...	$s_{182} = 182$
$H \in [158,159[$	$H \in [158,159[$		$H \in [145,146[$
$W \in [69,70[$	$W \in [68,69[$		$W \in [57,58[$
$0\ MGT$	$0\ MGT$		$0\ MGT$
$s_{183} = 183$	$s_{184} = 184$	...	$s_{364} = 364$
$H \in [158,159[$	$H \in [158,159[$		$H \in [145,146[$
$W \in [69,70[$	$W \in [68,69[$		$W \in [57,58[$
$8\ MGT$	$8\ MGT$		$8\ MGT$
$\vdots$	$\vdots$		$\vdots$
$s_{8\,009} = 8\,009$	$s_{8\,010} = 8\,010$	...	$s_{8\,190} = 8\,190\}$
$H \in [158,159[$	$H \in [158,159[$		$H \in [145,146[$
$W \in [69,70[$	$W \in [68,69[$		$W \in [57,58[$
$352\ MGT$	$352\ MGT$		$352\ MGT$

For the damage state space, the accumulated MGT variable is not considered. The assumption is that when the rail is damaged, there is no MGT value associated. Therefore, bearing in mind 13 width (W) and 14 height (H) states, there are 182 states with damage. The state space can be divided into 8 190 states without damage and 182 states with damage, performing a total of 8 372 states.

$$\underbrace{\begin{matrix} s=\{s_1=1 & s_2=2 & \dots & s_{182}=182 \\ H\in[158{,}159[& H\in[158{,}159[& & H\in[145{,}146[\\ W\in[69{,}70[& W\in[68{,}69[& & W\in[57{,}58[\\ 0\ MGT & 0\ MGT & & 0\ MGT \end{matrix}}_{Without\ Damage}$$

$$\underbrace{\begin{matrix} s_{183}=183 & s_{184}=184 & \dots & s_{364}=364 \\ H\in[158{,}159[& H\in[158{,}159[& & H\in[145{,}146[\\ W\in[69{,}70[& W\in[68{,}69[& & W\in[57{,}58[\\ 8\ MGT & 8\ MGT & & 8\ MGT \end{matrix}}_{Without\ Damage}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\underbrace{\begin{matrix} s_{8\,009}=8\,009 & s_{8\,010}=8\,010 & \dots & s_{8\,190}=8\,190 \\ H\in[158{,}159[& H\in[158{,}159[& & H\in[145{,}146[\\ W\in[69{,}70[& W\in[68{,}69[& & W\in[57{,}58[\\ 352\ MGT & 352\ MGT & & 352\ MGT \end{matrix}}_{Without\ Damage}$$

$$\underbrace{\begin{matrix} s_{8\,191}=8\,191 & s_{8\,192}=8\,192 & \dots & s_{8\,372}=8\,372\} \\ H\in[158{,}159[& H\in[158{,}159[& & H\in[145{,}146[\\ W\in[69{,}70[& W\in[68{,}69[& & W\in[57{,}58[\\ Damaged & Damaged & & Damaged \end{matrix}}_{With\ Damage}$$

A2 Definition of MDP state space for UIC60 rail profile

For UIC60 rail profile maintenance plan, the state space with the rail width (W) is divided into 16 different states.

$s = \{s_1 = 1$	$s_2 = 2$	...	$s_{16} = 16\}$
$W \in [71,72[$	$W \in [70,71[$		$W \in [56,57[$

Rail height (H) state space is divided into 16 different states. Therefore, the state space with the rail width (W) and height (H) is divided into 256 states, as shown below.

$s = \{s_1 = 1$	$s_2 = 2$	...	$s_{16} = 16$
$H \in [171,172[$	$H \in [171,172[$		$H \in [171,172[$
$W \in [71,72[$	$W \in [70,71[$		$W \in [56,57[$
$s_{17} = 17$	$s_{18} = 18$	...	$s_{32} = 32$
$H \in [170,171[$	$H \in [170,171[$		$H \in [170,171[$
$W \in [71,72[$	$W \in [70,71[$		$W \in [56,57[$
$\vdots$	$\vdots$		$\vdots$
$s_{241} = 241$	$s_{242} = 242$	...	$s_{256} = 256\}$
$H \in [156,157[$	$H \in [156,157[$		$H \in [156,157[$
$W \in [71,72[$	$W \in [70,71[$		$W \in [56,57[$

Besides rail height (H) and width (W), another important variable that must be considered to define the state space is the accumulated MGT, which is divided into 45 states, from 0 MGT to 352 MGT. Therefore, the state space with the rail width (W), height (H) and accumulated MGT is divided into 11 520 states, as shown below.

$s = \{s_1 = 1$	$s_2 = 2$	…	$s_{256} = 256$
$H \in [171,172[$	$H \in [171,172[$		$H \in [156,157[$
$W \in [71,72[$	$W \in [70,71[$		$W \in [56,57[$
$0\ MGT$	$0\ MGT$		$0\ MGT$
$s_{257} = 257$	$s_{258} = 258$	…	$s_{512} = 512$
$H \in [171,172[$	$H \in [171,172[$		$H \in [156,157[$
$W \in [71,72[$	$W \in [70,71[$		$W \in [56,57[$
$8\ MGT$	$8\ MGT$		$8\ MGT$
⋮	⋮		⋮
$s_{11\,265} = 11\,265$	$s_{11\,266} = 11\,266$	…	$s_{11\,520} = 11\,520\}$
$H \in [171,172[$	$H \in [171,172[$		$H \in [156,157[$
$W \in [71,72[$	$W \in [70,71[$		$W \in [56,57[$
$352\ MGT$	$352\ MGT$		$352\ MGT$

For the damage state space, the accumulated MGT variable is not considered. The assumption is that when the rail is damaged, there is no MGT value associated. Therefore, bearing in mind 16 width (W) and 16 height (H) states, there are 256 states with damage. The state space can be divided into 11 520 states without damage and 256 states with damage, performing a total of 11 776 states.

$$\underbrace{\begin{matrix} s=\{s_1=1 & s_2=2 & \dots & s_{256}=256 \\ H\in[171,172[& H\in[171,172[& & H\in[156,157[\\ W\in[71,72[& W\in[70,71[& & W\in[56,57[\\ 0\,MGT & 0\,MGT & & 0\,MGT \end{matrix}}_{Without\ Damage}$$

$$\underbrace{\begin{matrix} s_{257}=257 & s_{258}=258 & \dots & s_{512}=512 \\ H\in[171,172[& H\in[171,172[& & H\in[156,157[\\ W\in[71,72[& W\in[70,71[& & W\in[56,57[\\ 8\,MGT & 8\,MGT & & 8\,MGT \end{matrix}}_{Without\ Damage}$$

$$\begin{matrix} \vdots & \vdots & & \vdots \end{matrix}$$

$$\underbrace{\begin{matrix} s_{11\,265}=11\,265 & s_{11\,266}=11\,266 & \dots & s_{11\,520}=11\,520 \\ H\in[171,172[& H\in[171,172[& & H\in[156,157[\\ W\in[71,72[& W\in[70,71[& & W\in[56,57[\\ 352\,MGT & 352\,MGT & & 352\,MGT \end{matrix}}_{Without\ Damage}$$

$$\underbrace{\begin{matrix} s_{11\,521}=11\,521 & s_{11\,522}=11\,522 & \dots & s_{11\,776}=11\,776\} \\ H\in[171,172[& H\in[171,172[& & H\in[156,157[\\ W\in[71,72[& W\in[70,71[& & W\in[56,57[\\ Damaged & Damaged & & Damaged \end{matrix}}_{With\ Damage}$$

B1 Probability values of Markov Transition Matrices for the "Do Nothing" action for UIC54 rail profile

The transition probabilities to states without damage are written in expressions (5.3) - (5.6) and the transition probabilities to states with damage in expression (5.10).

For UIC54 rail profile MTM P_1^{54} ($8\,372 \times 8\,372$), $P_{Wear}{}^{(182\times182)}$ and $P_{Damage}{}^{(182\times182)}$ sub-matrices non-zero probabilities are defined as:

$$P_{Wear\ i,i}(MGT) = \left(1 - p_{Damage}(MGT)\right) \cdot \left(1 - p_{H54}(MGT)\right) \cdot \left(1 - p_{W54}(MGT)\right),$$

$$i \in \{1,2,\dots,168\} \backslash \{13,26,\dots,156\}$$

$$P_{Wear\ i,i}(MGT) = \left(1 - p_{Damage}(MGT)\right),$$

$$i \in \{\ 13,26,\dots,169,170,\dots,182\}$$

$$P_{Wear\ i,i+1}(MGT) = \left(1 - p_{Damage}(MGT)\right) \cdot \left(1 - p_{H54}(MGT)\right) \cdot p_{W54}(MGT),$$

$$i \in \{1,2,\dots,168\} \backslash \{13,26,\dots,156\}$$

$$P_{Wear\ i,i+13}(MGT) = \left(1 - p_{Damage}(MGT)\right) \cdot p_{H54}(MGT) \cdot \left(1 - p_{W54}(MGT)\right),$$

$$i \in \{1,2,\dots,168\} \backslash \{13,26,\dots,156\}$$

$$P_{Wear\ i,i+14}(MGT) = \left(1 - p_{Damage}(MGT)\right) \cdot p_{H54}(MGT) \cdot p_{W54}(MGT),$$

$$i \in \{1,2,\dots,168\} \backslash \{13,26,\dots,156\}$$

$$P_{Damage\ i,i}(MGT) = p_{Damage}(MGT),$$

$$i \in \{1,2,\dots,182\}$$

B2 Probability values of Markov Transition Matrices for the "Do Nothing" action for UIC60 rail profile

The transition probabilities to states without damage are written in expressions (5.3) - (5.6) and the transition probabilities to states with damage in expression (5.10).

For UIC60 rail profile MTM P_1^{60} $(11\ 776 \times 11\ 776)$, ${P_{Wear}}^{(256\times256)}$ and ${P_{Damage}}^{(256\times256)}$ sub-matrices non-zero probabilities are defined as:

$P_{Wear\ i,i}(MGT) = \left(1 - p_{Damage}(MGT)\right) \cdot \left(1 - p_{H60}(MGT)\right) \cdot \left(1 - p_{W60}(MGT)\right),$

$i \in \{1,2, \dots, 239\} \backslash \{16,32, \dots, 224\}$

$P_{Wear\ i,i}(MGT) = \left(1 - p_{Damage}(MGT)\right),$

$i \in \{\ 16, 32, \dots, 240,241, \dots, 256\}$

$P_{Wear\ i,i+1}(MGT) = \left(1 - p_{Damage}(MGT)\right) \cdot \left(1 - p_{H60}(MGT)\right) \cdot p_{W60}(MGT),$

$i \in \{1,2, \dots, 239\} \backslash \{16,32, \dots, 224\}$

$P_{Wear\ i,i+16}(MGT) = \left(1 - p_{Damage}(MGT)\right) \cdot p_{H60}(MGT) \cdot \left(1 - p_{W60}(MGT)\right),$

$i \in \{1,2, \dots, 239\} \backslash \{16,32, \dots, 224\}$

$P_{Wear\ i,i+17}(MGT) = \left(1 - p_{Damage}(MGT)\right) \cdot p_{H60}(MGT) \cdot p_{W60}(MGT),$

$i \in \{1,2, \dots, 239\} \backslash \{16,32, \dots, 224\}$

$P_{Damage\ i,i}(MGT) = p_{Damage}(MGT),$

$i \in \{1,2, \dots, 256\}$

C1 Probability values of Markov Transition Matrices for the "Grinding" action for UIC54 rail profile

For UIC54 rail profile MTM P_3^{54} $(8\,372 \times 8\,372)$, ${P_{PG}}^{(182\times182)}$ and ${P_{CG}}^{(182\times182)}$ sub-matrices probabilities are defined below.

Preventive grinding

For UIC54 rail profile, ${P_{PG}}^{(182\times182)}$ sub-matrix non-zero probabilities are defined as:

$P_{PG\ i,i} = (1 - p_{pg}),$

$i \in \{1,2, \dots ,169\}$

$P_{PG\ i,i} = 1,$

$i \in \{170,171, \dots ,182\}$

$P_{PG\ i,i+13} = p_{pg},$

$i \in \{1,2, \dots ,169\}$

Corrective grinding

For rail corrective grinding, a sub-matrix $P_{sub}^{(13\times13)}$ composed by the probabilities determined in 5.5.2 is defined:

$$P_{sub\,j}=\begin{bmatrix}
p_{cg\,j} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & p_{cg\,j} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & p_{cg\,j} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & p_{cg\,j} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & p_{cg\,j} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & p_{cg\,j} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & p_{cg\,j} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{cg\,j} & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{cg\,j} & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{cg\,j} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{cg\,j} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{cg\,j} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{cg\,j}
\end{bmatrix}$$

The sub-matrix $P_{CG}{}^{(182\times182)}$ is composed by the sub-matrix $P_{sub}^{(13\times13)}$ defined previously, as represented below:

$$P_{CG}=\begin{bmatrix}
P_{sub\,1} & P_{sub\,2} & P_{sub\,3} & P_{sub\,4} & P_{sub\,5} & P_{sub\,6} & P_{sub\,7} & P_{sub\,8} & P_{sub\,9} & P_{sub\,10} & 0 & 0 & 0 & 0\\
0 & P_{sub\,1} & P_{sub\,2} & P_{sub\,3} & P_{sub\,4} & P_{sub\,5} & P_{sub\,6} & P_{sub\,7} & P_{sub\,8} & P_{sub\,9} & P_{sub\,10} & 0 & 0 & 0\\
0 & 0 & P_{sub\,1} & P_{sub\,2} & P_{sub\,3} & P_{sub\,4} & P_{sub\,5} & P_{sub\,6} & P_{sub\,7} & P_{sub\,8} & P_{sub\,9} & P_{sub\,10} & 0 & 0\\
0 & 0 & 0 & P_{sub\,1} & P_{sub\,2} & P_{sub\,3} & P_{sub\,4} & P_{sub\,5} & P_{sub\,6} & P_{sub\,7} & P_{sub\,8} & P_{sub\,9} & P_{sub\,10} & 0\\
0 & 0 & 0 & 0 & P_{sub\,1} & P_{sub\,2} & P_{sub\,3} & P_{sub\,4} & P_{sub\,5} & P_{sub\,6} & P_{sub\,7} & P_{sub\,8} & P_{sub\,9} & P_{sub\,10}\\
0 & 0 & 0 & 0 & 0 & P_{sub\,1} & P_{sub\,2} & P_{sub\,3} & P_{sub\,4} & P_{sub\,5} & P_{sub\,6} & P_{sub\,7} & P_{sub\,8} & P_{sub\,9}+P_{sub\,10}\\
0 & 0 & 0 & 0 & 0 & 0 & P_{sub\,1} & P_{sub\,2} & P_{sub\,3} & P_{sub\,4} & P_{sub\,5} & P_{sub\,6} & P_{sub\,7} & P_{sub\,8}+\cdots+P_{sub\,10}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{sub\,1} & P_{sub\,2} & P_{sub\,3} & P_{sub\,4} & P_{sub\,5} & P_{sub\,6} & P_{sub\,7}+\cdots+P_{sub\,10}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{sub\,1} & P_{sub\,2} & P_{sub\,3} & P_{sub\,4} & P_{sub\,5} & P_{sub\,6}+\cdots+P_{sub\,10}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{sub\,1} & P_{sub\,2} & P_{sub\,3} & P_{sub\,4} & P_{sub\,5}+\cdots+P_{sub\,10}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{sub\,1} & P_{sub\,2} & P_{sub\,3} & P_{sub\,4}+\cdots+P_{sub\,10}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{sub\,1} & P_{sub\,2} & P_{sub\,3}+\cdots+P_{sub\,10}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{sub\,1} & P_{sub\,2}+\cdots+P_{sub\,10}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{sub\,1}+\cdots+p_{sub\,10}
\end{bmatrix}$$

C2 Probability values of Markov Transition Matrices for the "Grinding" action for UIC60 rail profile

For UIC60 rail profile MTM $P_3^{60}(11\,776 \times 11\,776)$, ${P_{PG}}^{(256\times256)}$ and ${P_{CG}}^{(256\times256)}$ sub-matrices probabilities are defined below.

Preventive grinding

For UIC60 rail profile, ${P_{PG}}^{(256\times256)}$ sub-matrix non-zero probabilities are defined as:

$P_{PG\ i,i} = (1 - p_{pg}),$

$i \in \{1,2, \dots ,240\}$

$P_{PG\ i,i} = 1,$

$i \in \{241,242, \dots ,256\}$

$P_{PG\ i,i+16} = p_{pg}\,,$

$i \in \{1,2, \dots ,240\}$

Corrective grinding

For rail corrective grinding, a sub-matrix $P_{sub}^{(16\times16)}$ composed by the probabilities determined in 5.5.2 is defined:

$$P_{sub\ j} = \begin{bmatrix}
p_{cg\ j} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & p_{cg\ j} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & p_{cg\ j} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & p_{cg\ j} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & p_{cg\ j} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & p_{cg\ j} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & p_{cg\ j} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{cg\ j} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{cg\ j} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{cg\ j} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{cg\ j} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{cg\ j} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{cg\ j} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{cg\ j} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{cg\ j} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{cg\ j}
\end{bmatrix}$$

The sub-matrix $P_{CG}{}^{(256\times256)}$ is composed by the sub-matrix $P_{sub}^{(16\times16)}$ defined previously, as represented below:

$$P_{CG} = \begin{bmatrix}
P_{sub\ 1} & P_{sub\ 2} & P_{sub\ 3} & P_{sub\ 4} & P_{sub\ 5} & P_{sub\ 6} & P_{sub\ 7} & P_{sub\ 8} & P_{sub\ 9} & P_{sub\ 10} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & P_{sub\ 1} & P_{sub\ 2} & P_{sub\ 3} & P_{sub\ 4} & P_{sub\ 5} & P_{sub\ 6} & P_{sub\ 7} & P_{sub\ 8} & P_{sub\ 9} & P_{sub\ 10} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & P_{sub\ 1} & P_{sub\ 2} & P_{sub\ 3} & P_{sub\ 4} & P_{sub\ 5} & P_{sub\ 6} & P_{sub\ 7} & P_{sub\ 8} & P_{sub\ 9} & P_{sub\ 10} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & P_{sub\ 1} & P_{sub\ 2} & P_{sub\ 3} & P_{sub\ 4} & P_{sub\ 5} & P_{sub\ 6} & P_{sub\ 7} & P_{sub\ 8} & P_{sub\ 9} & P_{sub\ 10} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & P_{sub\ 1} & P_{sub\ 2} & P_{sub\ 3} & P_{sub\ 4} & P_{sub\ 5} & P_{sub\ 6} & P_{sub\ 7} & P_{sub\ 8} & P_{sub\ 9} & P_{sub\ 10} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & P_{sub\ 1} & P_{sub\ 2} & P_{sub\ 3} & P_{sub\ 4} & P_{sub\ 5} & P_{sub\ 6} & P_{sub\ 7} & P_{sub\ 8} & P_{sub\ 9} & P_{sub\ 10} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & P_{sub\ 1} & P_{sub\ 2} & P_{sub\ 3} & P_{sub\ 4} & P_{sub\ 5} & P_{sub\ 6} & P_{sub\ 7} & P_{sub\ 8} & P_{sub\ 9} & P_{sub\ 10} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{sub\ 1} & P_{sub\ 2} & P_{sub\ 3} & P_{sub\ 4} & P_{sub\ 5} & P_{sub\ 6} & P_{sub\ 7} & P_{sub\ 8} & P_{sub\ 9} + P_{sub\ 10} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{sub\ 1} & P_{sub\ 2} & P_{sub\ 3} & P_{sub\ 4} & P_{sub\ 5} & P_{sub\ 6} & P_{sub\ 7} & P_{sub\ 8} + \cdots + P_{sub\ 10} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{sub\ 1} & P_{sub\ 2} & P_{sub\ 3} & P_{sub\ 4} & P_{sub\ 5} & P_{sub\ 6} & P_{sub\ 7} + \cdots + P_{sub\ 10} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{sub\ 1} & P_{sub\ 2} & P_{sub\ 3} & P_{sub\ 4} & P_{sub\ 5} & P_{sub\ 6} + \cdots + P_{sub\ 10} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{sub\ 1} & P_{sub\ 2} & P_{sub\ 3} & P_{sub\ 4} & P_{sub\ 5} + \cdots + P_{sub\ 10} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{sub\ 1} & P_{sub\ 2} & P_{sub\ 3} & P_{sub\ 4} + \cdots + P_{sub\ 10} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{sub\ 1} & P_{sub\ 2} & P_{sub\ 3} + \cdots + P_{sub\ 10} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{sub\ 1} & P_{sub\ 2} + \cdots + P_{sub\ 10} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{sub\ 1} + \cdots + p_{sub\ 10}
\end{bmatrix}$$